

***BHARTIYA INSTITUTE OF ENGINEERING & TECHNOLOGY  
SIKAR***

***DEPARTMENT OF CIVIL ENGINEERING***



***LAB MANUAL***

***6CE4-25 : FOUNDATION ENGINEERING***

***BY: MR AMINUDDIN***

***BHARTIYA INSTITUTE OF ENGINEERING & TECHNOLOGY  
SIKAR***

***DEPARTMENT OF CIVIL ENGINEERING***

**SYLLABUS**

**SUBJECT: FOUNDATION ENGINEERING (6CE4-25)**

1. Design of isolated shallow footings, combined footings, raft foundations.
2. Design of pile foundations.
3. Design of wells and cassions.
4. Design of machine foundation.
5. Design of retaining structures



## 15.1. INTRODUCTION

Any structure is generally considered to have two main portions (i) the *superstructure* and (ii) the *substructure*. The substructure transmits the loads of superstructure to the supporting soil and is generally termed as the *foundation*. *Footing* is that portion of the foundation which ultimately delivers the load to the soil, and is thus in contact with it. The load of the superstructure is transmitted to the foundation or structure through either columns or walls. The object of providing foundation to a structure is to distribute the load to the soil in such a way that the maximum pressure on the soil does not exceed its permissible bearing value, and at the same time the settlement is within the permissible limits.

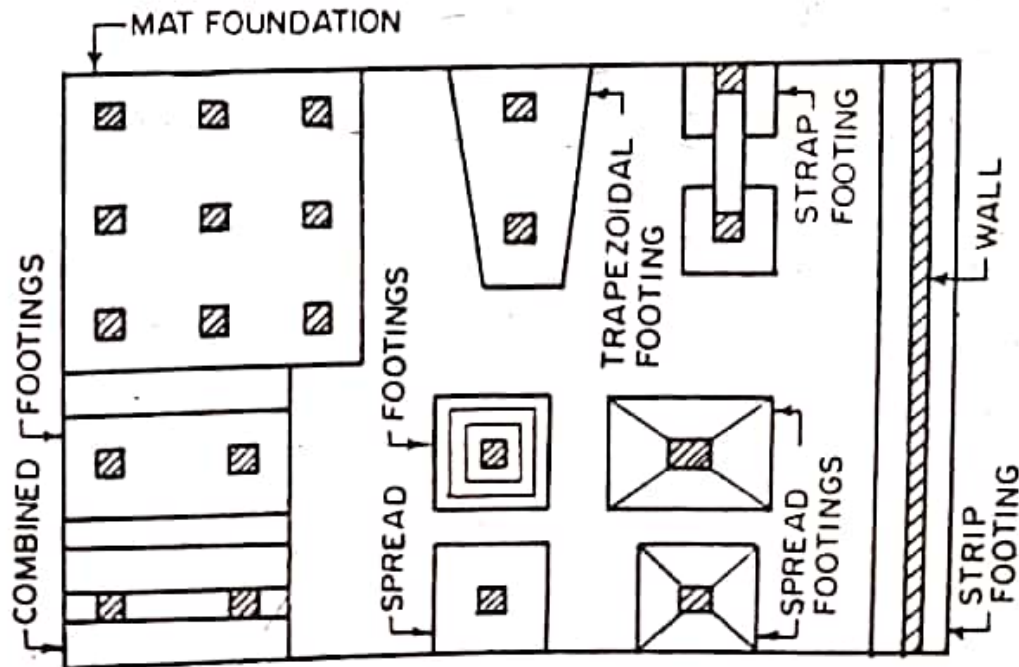


FIG. 15.1 VARIOUS TYPES OF SHALLOW FOOTINGS.

Foundations may be broadly classified under two heads : *shallow foundation and deep foundation*. According to Terzaghi, a foundation is shallow if its depth is equal to or less than its width. In the case of deep foundation, the depth is equal to or greater than the width. Apart from deep strip, rectangular or square foundations, other common forms of deep foundations are : *pier foundation, pile foundation and well foundation*. The shallow foundations are of the following types : Spread footing, strap footing, combined footing and mat or raft footing. Fig. 15.1 shows the common types of shallow foundations.

**Spread footings.** A spread footing or simply footing, is a type of shallow foundation used to transmit the load of an isolated column, or that of a wall, on the subsoil. In the case of wall, the footing is *continuous* while in the case of column, it is *isolated*. Fig. 15.2 shows some common types of reinforced concrete spread footing.

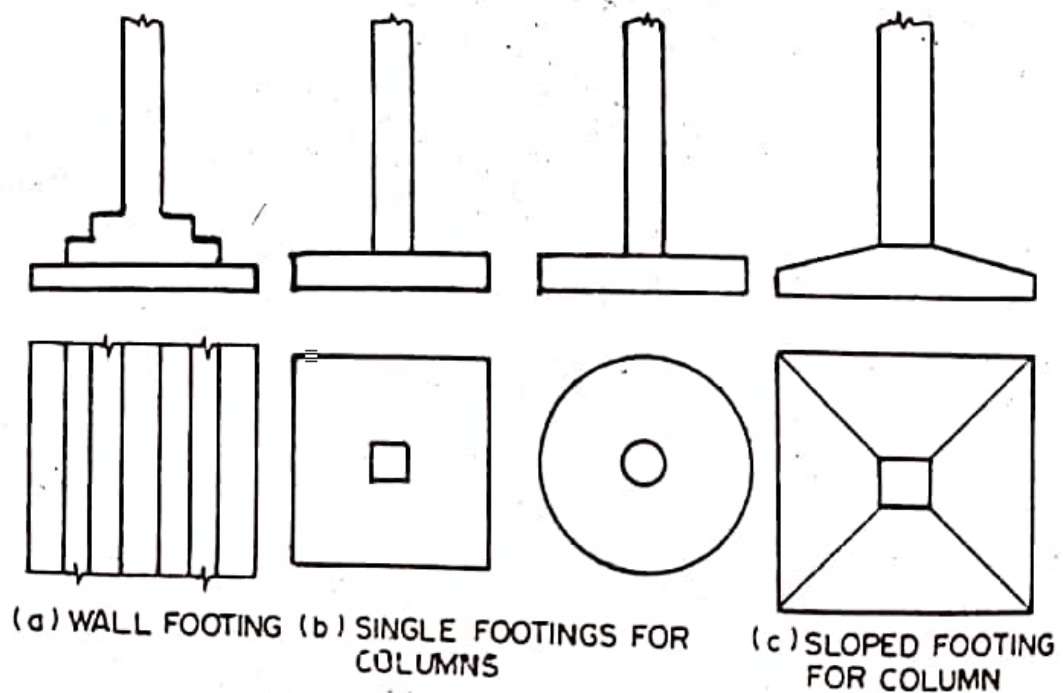


FIG. 15.2. TYPICAL SPREAD FOOTINGS.

**Combined footings.** A spread footing which supports two or more columns is termed as a *combined footing*. Such a footing, is provided when the individual footings are either very near to each other, or overlap. Combined footings may either be rectangular or trapezoidal (Fig. 15.2).

**Strap or cantilever footing.** A strap footing consists of spread footings of two columns connected by a strap beam. The strap beam does not remain in contact with soil, and thus does not transfer

any pressure to the soil. Such a footing is generally used to combine the footing of outer column to the adjacent one so that the footing of the former does not extend in the adjoining property [Fig. 15.3(c)].

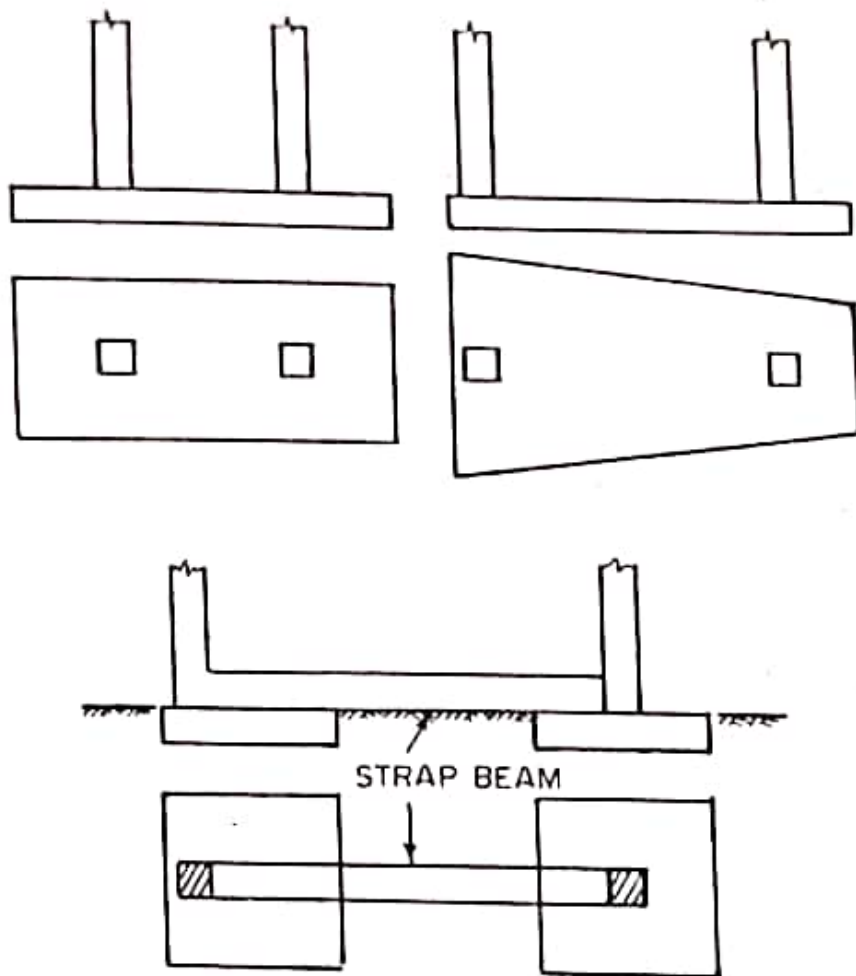


FIG. 15.3.

**Mat or raft foundation.** A mat or raft is a combined footing that covers the entire area beneath a structure and supports all the walls and columns. When the available soil pressure is low or the building loads are heavy, the use of spread footings would cover more than one-half of the area and it may prove more economical to use mat or raft foundation.

**Pile foundation.** Pile foundation is a deep foundation used where the top soil is relatively weak. Piles transfer the load to a lower stratum of greater bearing capacity, by way of end bearing, or to the intermediate soil through skin friction. This is most common type of deep foundation generally used for buildings where a group of piles transfer the load of the super-structure to the sub-soil.

## 15.2. PRESSURE DISTRIBUTION BENEATH FOOTINGS

Both from observations as well as the analytical studies from theory of elasticity, it is known that the pressure distribution beneath footings, symmetrically loaded is not uniform. The pressure intensities depend upon the rigidity of footing, soil type, and the condition of soil. Fig. 15.4(a) and (b) show the probable pressure distribution beneath a rigid footing on a loose cohesionless soil and cohesive soil. Fig. 15.4(c) shows the usually assumed uniform pressure distributions.

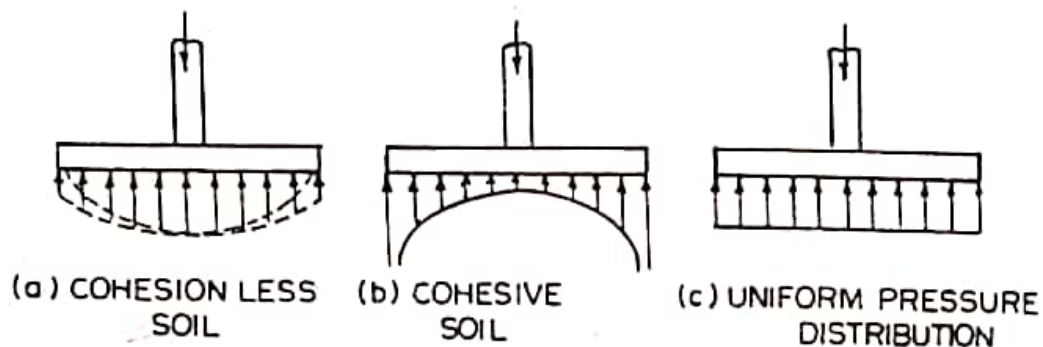


FIG. 15.4. PRESSURE DISTRIBUTION BENEATH FOOTINGS.

When a rigid footing rests on cohesionless soil, the soil grain at the outer edge have no lateral restraint, whereas in centre the soil is relatively confined resulting in a pressure distribution shown in Fig. 15.4(a). For the footings on cohesive soils, the edge stresses may be very large. However, the pressure distribution may be considered as *linear* as shown in Fig. 15.4(c), for the purpose of the design of reinforced concrete footings. Once the pressure distribution is known, the bending moments and shear force in the footing can be calculated, and the thickness of the structural member of the footing, alongwith the reinforcement etc., can be calculated using the usual principles of reinforced concrete.

**Example 15.4.** Design a rectangular isolated footing of uniform thickness for R.C. column bearing a vertical load of 600 kN, and having a base size of 400 × 600 mm. The safe bearing capacity of soil may be taken as 120 kN/m<sup>2</sup>. Use M 15 concrete. Take  $\sigma_{st} = 140 \text{ N/mm}^2$ .

**Solution**

1. *Design constants*

For M 15 concrete, and for  $\sigma_{st} = 140$ , we have  
 $k = 0.404$  ;  $j = 0.865$  and  $R = 0.874$

2. *Size of footing*

$$W = 600 \text{ kN}$$

Let  $W'$  be equal to 10%  $W = 60 \text{ kN}$ .

$$\therefore A = \frac{660}{120} = 5.5 \text{ m}^2$$

$$\text{Let ratio of } B \text{ to } L = \frac{40}{60} = \frac{2}{3}$$

$$\therefore \frac{2}{3}L \times L = 5.5$$

$$\text{or } L = 2.9 \text{ m.}$$

$$\therefore B = \frac{2}{3} \times 2.9 \text{ m}$$

However, provide a footing of size 2 m × 3 m.

$$\text{Net upward pressure } p_0 = \frac{600}{2 \times 3} = 100 \text{ kN/m}^2$$

3. *Design of section*

Refer Fig. 15.14.

Bending moment  $M_1$  about section X-X is given by

$$\begin{aligned} M_1 &= \frac{p_0 B}{8} (L - a)^2 \text{ kN-m} \\ &= \frac{100 \times 2}{8} (3 - 0.6)^2 \times 10^6 \text{ N-mm} \end{aligned}$$

$$= 144 \times 10^6 \text{ N-mm}$$

$$\therefore d = \sqrt{\frac{M_1}{RB}} = \sqrt{\frac{144 \times 10^6}{0.874 \times 2000}}$$

$$= 287 \text{ mm.}$$

Keep  $d = 290$  mm and total depth = 350 mm.

Provide uniform thickness for the entire footing.

B.M.  $M_2$  about section Y-Y is given by

$$M_2 = \frac{p_0 L}{8} (B - b)^2 \times 10^6 \text{ N-mm}$$

$$= \frac{100 \times 3}{8} (2 - 0.4)^2 \times 10^6 = 96 \times 10^6 \text{ N-mm}$$

Thus,  $M_2 < M_1$ .

The effective depth found above has to be checked for shear.

#### 4. Check for shear

For the *beam action* total S.F. along section AB [Fig. 15.10(a)]

is

$$V = p_0 B \left( \frac{L}{2} - \frac{a}{2} - d \right) = p_0 B \left( \frac{L - a}{2} - d \right)$$

$$= 100 \times 2 \left( \frac{3 - 0.6}{2} - 0.29 \right) = 182 \text{ kN}$$

$$= 170000 \text{ N.}$$

$$\tau_v = \frac{V}{Bd} = \frac{182000}{2000 \times 290} = 0.314 \text{ N/mm}^2.$$

For M 15 concrete,  $p = 0.72\%$  and hence  $\tau_c = 0.33 \text{ N/mm}^2$

Also for  $D > 300$  mm,  $k = 1$  from Table 3.2.

$$\therefore \tau_v < \tau_c \text{ (safe).}$$

For the *two way action or punching shear action* along ABCD [Fig. 16.10(b)],

$$\text{Perimeter } ABCD = 2 \{ (a + d) + (b + d) \}$$

$$= 2 \{ (0.6 + 0.29) + (0.4 + 0.29) \}$$

$$= 2 \{ 0.89 + 0.69 \} = 3.16 \text{ m}$$

$$= 3160 \text{ mm.}$$

$$\text{Area } ABCD = 0.89 \times 0.69 = 0.6141 \text{ m}^2$$

$$\therefore \text{Punching shear} = 100[(2 \times 3) - 0.6141]$$

$$= 538.59 \text{ kN}$$

$$\therefore \tau_v = \frac{538.59 \times 1000}{3160 \times 290} = 0.59 \text{ N/mm}^2.$$



Allowable shear stress  $\tau_c$  is given by

$$\tau_c = 0.16 \sqrt{f_{ck}} = 0.16 \sqrt{15} \approx 0.62 \text{ N/mm}^2.$$

$$k_s = (0.5 + \beta_c) = \left(0.5 + \frac{0.4}{0.6}\right) = 1.17$$

However, adopt max.  $k_s = 1$ .

$$\therefore k_s \tau_c = 1 \times 0.62 = 0.62 \text{ N/mm}^2.$$

This is more than  $\tau_v = 0.59 \text{ N/mm}^2$ . Hence safe.

Thus the effective depth  $d = 290 \text{ mm}$  is alright.

### 5. Design for reinforcement

Area  $A_{st1}$  of long bars calculated for moment  $M_1$  is given by

$$A_{st1} = \frac{M_1}{\sigma_{st} j d} = \frac{144 \times 10^6}{140 \times 0.865 \times 290} = 4100 \text{ mm}^2$$

Using 12 mm  $\Phi$  bars,  $A_\Phi = 113 \text{ mm}^2$

$$\therefore \text{No. of bars} = \frac{4100}{113} \approx 36.$$

These are to be distributed uniformly in a width  $B = 2 \text{ m}$ .

Effective depth for top layer of reinforcement  $= 290 - 12 = 278 \text{ mm}$ .

The area  $A_{st2}$  of short bars calculated for  $M_2$  is given by

$$A_{st2} = \frac{M_2}{\sigma_{st} j d} = \frac{97 \times 10^6}{140 \times 0.865 \times 278} = 2880 \text{ mm}^2.$$

This area is to be provided in two distinct band widths. Area

$A_{st2(B)}$  in central bend of width  $B = 2 \text{ m}$  is given by

$$A_{st2(B)} = \frac{2A_{st2}}{\beta + 1} = \frac{2 \times 2880}{\frac{3}{2} + 1} = 2304 \text{ mm}^2.$$

$$\therefore \text{No. of 12 mm } \Phi \text{ bars} = \frac{2304}{113} \approx 20 \text{ to be provided in}$$

central bend width  $= 2 \text{ m}$ .

Remaining area in each end band strip

$$= \frac{1}{2} (2880 - 2304) = 288 \text{ mm}^2.$$

$$\text{No. of 12 mm } \Phi \text{ bars} = \frac{288}{113} \approx 3, \text{ to be provided in each}$$

end band of width  $\frac{1}{2} (L - B) = \frac{1}{2} (3 - 2) = 0.5 \text{ m}$ .

### 6. Test for development length

$$L_d = \frac{\sigma_{st}}{4\tau_{bd}} \Phi = \frac{140}{4 \times 0.6} \Phi = 58.3 \Phi$$

$$= 58.3 \times 12 = 700 \text{ mm.}$$

Providing 60 mm side cover, length available

$\frac{1}{2}[B - b] - 60 = \frac{1}{2}[2000 - 400] - 60 = 740 \text{ mm,}$   
 which is greater than  $L_d$ . Hence O.K.

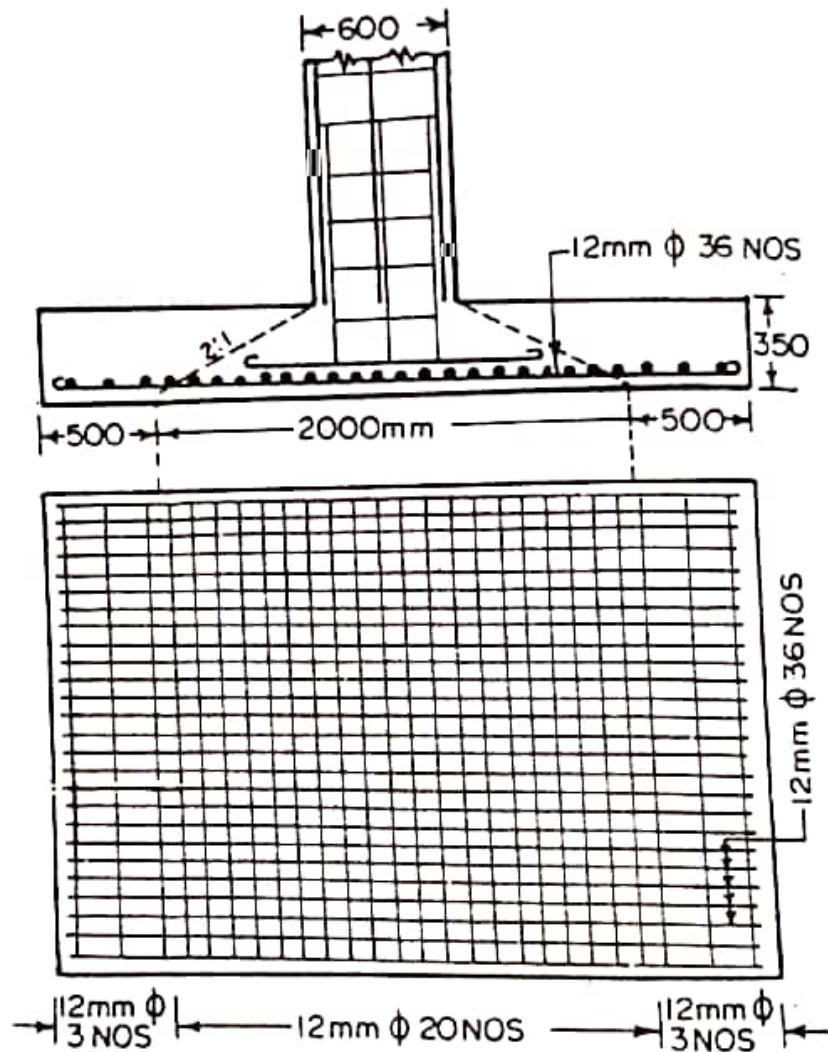


FIG. 15.25

7. Check for transfer of load at the base

$$A_2 = 600 \times 600 = 360000 \text{ mm}^2$$

At a rate of spread of 2:1,

$$A_1 = [600 + 2(2 \times 350)]^2 = (2000)^2 = 4 \times 10^6 \text{ mm}^2$$

$$\therefore \sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{4 \times 10^6}{360000}} = 3.33 > 2$$

Adopt max. value of  $\sqrt{A_1/A_2}$  as 2.

$$\begin{aligned} \therefore \text{permissible bearing stress} &= 0.25 f_{ck} \sqrt{\frac{A_1}{A_2}} \\ &= 0.25 \times 15 \times 2 = 7.5 \text{ N/mm}^2. \end{aligned}$$

**Design Example 16.1** *Design combined rectangular footing for two columns A and B, carrying loads of 500 and 700 kN respectively. Column A is 300 mm × 300 mm in size and column B is 400 mm × 400 mm in size. The centre to centre spacing of the columns is 3.4 metres.*

The safe bearing capacity of soil may be taken as  $150 \text{ N/m}^2$ . Use M 15 mix. Take  $\sigma_{st} = 140 \text{ N/mm}^2$ .

**Solution.**

**1. Design constants**

For M 15 mix. and  $\sigma_{st} = 140 \text{ N/mm}^2$ , we have

$$k = 0.404 ; j = 0.865 \text{ and } R = 0.874$$

**2. Size of footing**

$$W_1 = 500 \text{ kN and } W_2 = 700 \text{ kN}$$

Let the weight of footing =  $W' = 10\%$  of  $(W_1 + W_2) = 120 \text{ kN}$

$$\therefore A = \frac{500 + 700 + 120}{150} = 8.8 \text{ m}^2$$

Let the size of the footing be  $1.8 \text{ m} \times 5 \text{ m}$

The projections  $a_1$  and  $a_2$  should be such that C.G. of footing coincides with the C.G. of column loads. The distance  $\bar{x}$  of the C.G. of column loads from centre of column A is given by

$$\bar{x} = \frac{W_2 l}{W_1 + W_2} = \frac{700 \times 3.4}{500 + 700} \approx 2 \text{ m}$$

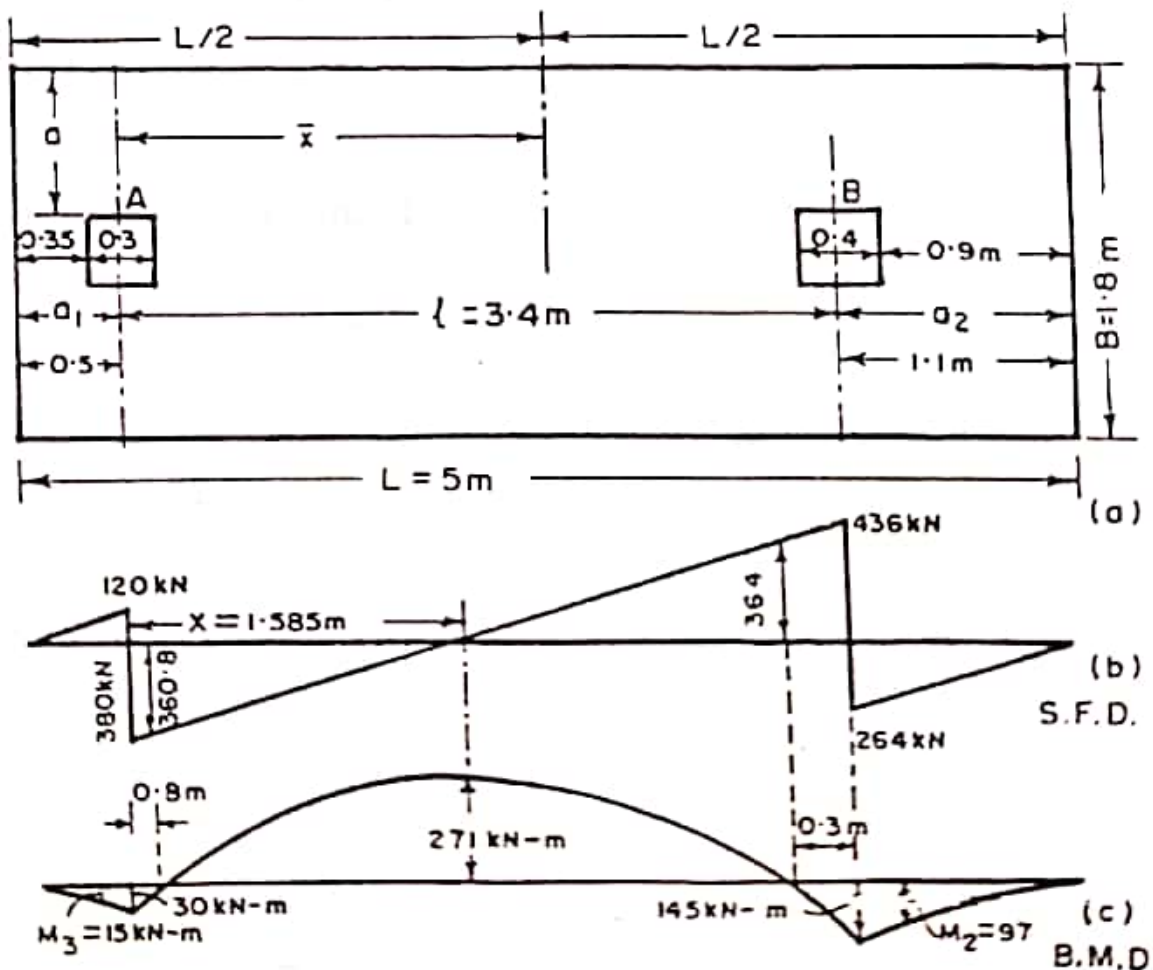


FIG. 16.6

$$\therefore a_1 + \bar{x} = \frac{L}{2}$$

$$\text{or } a_1 = \frac{L}{2} - \bar{x} = \frac{5}{2} - 2 = 0.5 \text{ m}$$

$$a_2 = L - (l + a_1) = 5 - (3.4 + 0.5) = 1.1 \text{ m}$$

The net upward pressure  $p_0$  is given by

$$p_0 = \frac{W_1 + W_2}{B \times L} = \frac{500 + 700}{1.8 \times 5} = \frac{400}{3} \text{ kN/m}^2$$

$\therefore$  pressure  $w$  per metre length

$$= p_0 B = \frac{400}{3} \times 1.8 = 240 \text{ kN/m}^2$$

### 3. B.M. and S.F. diagrams

The B.M. and S.F. diagrams in the longitudinal directions are shown in Fig. 16.6.

$$\text{S.F. just to the left of } A = 240 \times 0.5 = 120 \text{ kN}$$

$$\text{S.F. just to the right of } A = 500 - 120 = 380 \text{ kN}$$

$$\text{S.F. just to the right of } B = 240 \times 1.1 = 264 \text{ kN}$$

$$\text{S.F. just to the left of } B = 700 - 264 = 436 \text{ kN}$$

S.F. will be zero at distance  $x$  from  $A$ , its magnitude being given by  $x = \frac{380}{240} = 1.585 \text{ m}$ .

The maximum hogging B.M.  $M_1$  will therefore occur at this section, its magnitude being given by

$$M_1 = (500 \times 1.585) - \frac{240}{2} (0.5 + 1.585)^2$$

$$\approx 271 \text{ kN-m} = 2.71 \times 10^8 \text{ N-mm}$$

$$\text{Sagging B.M. at } B = \frac{240 (1.1)^2}{2} = 145 \text{ kN-m} = 1.45 \times 10^8 \text{ N-mm}$$

$$\text{Sagging B.M. at } A = \frac{240}{2} (0.5)^2 = 30 \text{ kN-m} = 0.3 \times 10^8 \text{ N-mm}$$

Sagging B.M. at the outer face of column  $B$  is

$$M_2 = \frac{240}{2} (0.9)^2 = 97 \text{ kN-m} = 0.97 \times 10^8 \text{ N-mm}$$

Sagging B.M. at the outer face of column  $A$  is

$$M_3 = \frac{240}{2} (0.35)^2 = 15 \text{ kN-m} = 0.15 \times 10^8 \text{ N-mm}$$

Let the point of contraflexure occur at  $x$  from the centre of column  $A$ .

$$\text{Then } M_x = 500x - \frac{240}{2}(x + 0.5)^2 = 0$$

which gives  $x = 0.08$  or  $3.08$  m

Hence first point of contraflexure occurs at  $0.08$  m to the right centre of column  $A$  and the second one occurs at  $3$  m from  $A$ , or  $3.4 - 3.1 = 0.3$  m to the left of centre of column  $B$ .

$$\begin{aligned} \text{S.F. at the first point of contraflexure} \\ = 380 - (240 \times 0.08) = 360.8 \text{ kN} \end{aligned}$$

$$\begin{aligned} \text{S.F. at the second point of contraflexure} \\ = 436 - (240 \times 0.3) = 364 \text{ kN} \end{aligned}$$

#### 4. Effective depth from bending compression

The effective depth  $d$  for hogging B.M.  $M_1$  is given by

$$d = \sqrt{\frac{2.71 \times 10^8}{0.874 \times 1800}} = 416 \text{ mm}$$

#### 5. Check for punching shear

Let us check the above depth for punching shear, for which the critical plane lies at  $d/2 = 208$  mm around column  $B$ . Width  $b_o$  of critical plane  $= 400 + 208 = 608$  mm.

$$\begin{aligned} \text{Punching shear force at column } B &= 700 - \frac{400}{3} \times (0.608)^2 \\ &\approx 651 \text{ kN.} \end{aligned}$$

$$\therefore \text{ Shear stress} = \frac{651 \times 1000}{4 \times 608 \times 416} = 0.643 \text{ N/mm}^2.$$

$$\begin{aligned} \text{Permissible shear stress} &= k_s \tau_c = 1 \times 0.16 \sqrt{f_{ck}} \\ &= 1 \times 0.16 \sqrt{15} = 0.62 \text{ N/mm}^2 \end{aligned}$$

This is less than the actual shear stress. Hence the depth found from bending compression is inadequate.

$$\text{The required depth } d = \frac{651 \times 1000}{4 \times 608 \times 0.62} = 432 \text{ mm.}$$

Using  $60$  mm cover to the centre of reinforcement,  $D = 432 + 60 = 492$  mm.

However, provide  $D = 500$  mm so that available  $d = 500 - 60 = 440$  mm.

#### 6. Design for bending tension in longitudinal direction

(i) Reinforcement for hogging bending moment  $M_1$

$$A_{st} = \frac{M_1}{\sigma_{st} j d} = \frac{2.71 \times 10^8}{140 \times 0.865 \times 440} = 5086 \text{ mm}^2$$

Using 16 mm  $\phi$  bars, having  $A_{\phi} = \frac{\pi}{4} (16)^2 = 201 \text{ mm}^2$

$$\therefore \text{No. of bars} = \frac{5086}{201} \approx 25.3 \approx 26.$$

Thus provide 26 bars uniformly distributed over the width of 1.8 m over top face.

At the point of contraflexure near B, shear force  $F = 364 \text{ kN}$ .

Let  $n$  = no. of 16 mm  $\phi$  bars required at the point of contraflexure, from the point of requirements of Code provisions inherent in the equation

$$\frac{M_1}{V} + L_0 \geq L_d$$

where  $M_1$  = moment of resistance of the section =  $A_{st} \sigma_{st} j d$   
 $= n \times 201 \times 140 \times 0.865 \times 440$   
 $= n \times 10.71 \times 10^6 \text{ N-mm}$

$V$  = S.F. at point of contraflexure =  $364 \times 10^3 \text{ N}$

$L_0$  =  $d$  for 12  $\phi$  whichever is greater  
 $= 440$  or  $(12 \times 16)$  which is greater  
 $= 440 \text{ mm}$

$$L_d \approx 58.3 \phi = 58.3 \times 16 = 933 \text{ mm}$$

$$\therefore \frac{M_1}{V} + L_0 \geq L_d$$

$$\text{or } \frac{n \times 10.71 \times 10^6}{364 \times 10^3} + 440 = 933$$

From which  $n = 16.8 \approx 17$

However, all the 26 bars are taken up to the outer faces of both the columns. Out of these, 18 bars are stopped and 8 bars are taken straight upto either edge of footing. These bars will serve as anchorage for the stirrups.

(ii) Reinforcement for sagging B.M.  $M_2$  is given by

$$A_{st2} = \frac{M_2}{\sigma_{st} j d} = \frac{0.97 \times 10^8}{140 \times 0.865 \times 440} = 1820 \text{ mm}^2$$

Using 12 mm  $\phi$  bars, No. of bars =  $\frac{1820}{113} = 16.1 \approx 17$

These no. of bars should be sufficient from development length point of view so as to satisfy the following criterion at the point of contraflexure near the inner face of column B.

$$\frac{M_1}{V} + L_0 \geq L_d$$

Let  $n$  = number of bars required to satisfy this criterion

$$\begin{aligned} \therefore M_1 &= A_{st} \sigma_{st} j d = (n \times 113) 140 \times 0.865 \times 440 \\ &= n \times 6.02 \times 10^6 \text{ N-mm} \end{aligned}$$

$$V = \text{S.F.} = 364 \text{ kN} = 3.64 \times 10^5 \text{ N}$$

$$L_0 = d \text{ or } 12 \phi \text{ whichever is more} = 440 \text{ mm.}$$

$$L_d = 58.3 \phi = 58.3 \times 12 = 700 \text{ mm.}$$

Substituting the values, we get

$$\frac{n \times 6.02 \times 10^6}{3.69 \times 10^5} + 440 = 700$$

which gives  $n = 15.7 \approx 16$ .

Hence 17 bars of 12 mm  $\phi$  will be sufficient.

These bars should be extended by  $l_o$  (=440 mm) beyond the point of contraflexure. At this point curtail 9 bars and continue 8 bars throughout the length to serve as anchor bars for stirrups.

(iii) Reinforcement for sagging B.M.  $M_3$

$$A_{st3} = \frac{0.15 \times 10^8}{140 \times 0.865 \times 440} = 282 \text{ mm}^2$$

Minimum reinforcement @ 0.15% of cross-sectional area

$$= \frac{0.15 \times 500 \times 1800}{100} = 1350 \text{ mm}^2$$

$$\therefore \text{No. of 12 mm } \phi \text{ bars} = \frac{1350}{113} = 12$$

Let us find the reinforcement from development length point of view so that the following relation is satisfied at the point of contraflexure near the inner face of column A :

$$\frac{M_1}{V} + L_0 \geq L_d$$

Let  $n$  = number of bars required to satisfy the above criterion

$$\begin{aligned} M_1 &= A_{st} \sigma_{st} j d = (n \times 113) \times 140 \times 0.865 \times 440 \\ &= n \times 6.02 \times 10^6 \text{ N-mm} \end{aligned}$$

$$\begin{aligned} V &= \text{shear force at point of contraflexure} \\ &= 360.8 \text{ kN} = 360.8 \times 10^3 \text{ N} \end{aligned}$$

$$L_0 = 12 \phi \text{ or } d \text{ whichever is more} = 440 \text{ mm}$$

$$L_d = 58.3 \phi = 58.3 \times 12 = 700 \text{ mm.}$$



Substituting the values,

$$\frac{n \times 6.02 \times 10^6}{360.8 \times 10^3} + 440 = 700$$

From which  $n = 15.6 \approx 16$ .

Continue these upto 440 mm beyond the point of contraflexure. At this point, curtail 9 bars and continue remaining 8 bars to serve as anchor bars for shear stirrups.

#### 6. Check for one way shear

In the cantilever portion, test for one way shear (diagonal tension) is made at a distance  $d = 0.44$  m from the column face or at  $0.44 + 0.20 = 0.64$  m from the centre of column  $B$ . Shear  $V = 264 - 240 \times 0.64 = 110.4$  kN.

$$\therefore \tau_v = \frac{110.4 \times 1000}{1800 \times 440} = 0.14 \text{ N/mm}^2$$

Permissible shear stress  $= k \tau_c = \tau_c = 0.33 \text{ N/mm}^2$  for  $p = 0.72\%$  (balanced section), obtained from Table 3.1.

Hence safe.

Similarly, in the cantilever portion near column  $A$ , the footing will be safe in shear.

For diagonal tension between  $A$  and  $B$ , near column  $B$ , crack can occur at the bottom face of the footing (*i.e.* for sagging B.M.) at a distance of  $d = 440$  mm, or at the top of footing (*i.e.*, for the hogging B.M.) at the point of contraflexure distant 300 mm from the centre of the column, whichever is nearer. Shear force at the point of contraflexure  $= 364$  kN which is more.

$$\therefore \tau_v = \frac{364 \times 1000}{1800 \times 440} = 0.46 \text{ N/mm}^2$$

which is more than the permissible shear stress  $\tau_c = 0.33 \text{ N/mm}^2$ . Hence shear reinforcement is necessary.

Using 12 mm  $\phi$  8-legged stirrups  $A_{sv} = 8 \times 113 = 904 \text{ mm}^2$

$$\text{Spacing } s_v = \frac{\sigma_{sv} A_{sv} d}{V_s} = \frac{140 \times 904 \times 440}{364 \times 1000} \\ = 153 \text{ mm}$$

Hence provide there @ 150 mm c/c

$$\text{The S.F. for which stress is } 0.33 \text{ N/mm}^2 \text{ is } = \frac{0.33}{0.46} \times 364 \\ = 261 \text{ kN}$$

This occurs at a distance of  $\frac{436 - 261}{240} = 0.73$  m from the centre of the column  $B$  or 0.53 m from the inner face of the column. Hence provide at least 5 stirrups @ 150 mm c/c, for a distance of 0.6 m from the face of the column  $B$ .

Similarly, near column  $A$ , tension crack can occur in the hogging portion at the inner face of the column since the point of contraflexure happens to fall under the column. S.F. at the inner face =  $380 - 240 \times 0.15 = 344$  kN

$$\therefore \tau_v = \frac{344 \times 1000}{1800 \times 440} = 0.434 \text{ N/mm}^2$$

against a permissible value of  $0.33 \text{ N/mm}^2$ . Hence shear reinforcement is necessary.

Hence provide 5 Nos. 12 mm  $\Phi$  8-legged stirrups @ 150 mm c/c, for a distance of 600 mm from the inner face of the column.

Spacing of nominal stirrups is given by

$$s_v = \frac{2.5 A_{sv} f_v}{b} = \frac{2.5 \times 904 \times 250}{1800} = 314 \text{ mm}$$

Hence for the rest of the length, provide 12 mm  $\Phi$  8-lgd nominal stirrups @ 300 mm c/c.

### 7. Transverse reinforcement

The footing will bend transversely near each column face. Projection  $a$  beyond the face of column  $A = \frac{1}{2}(1800 - 300) = 750$  mm. Width  $B_1$  of bending strip =  $b + 2d = 300 + 2 \times 440 = 1180$  mm. However, width available to the left of outer face of column  $A = 500 - 150 = 350$  mm only instead of 440 mm. Hence available  $B_1 = 350 + 300 + 440 = 1090$  mm = 1.09 m.

$$\text{Net upward pressure } p_0' = \frac{W_i}{B \times B_1} = \frac{500}{1.8 \times 1.09} = 254.8 \text{ kN/m}^2$$

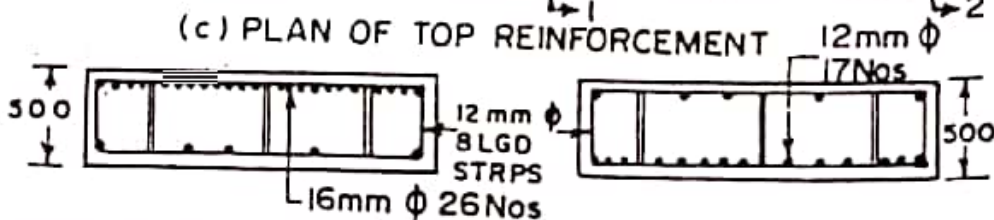
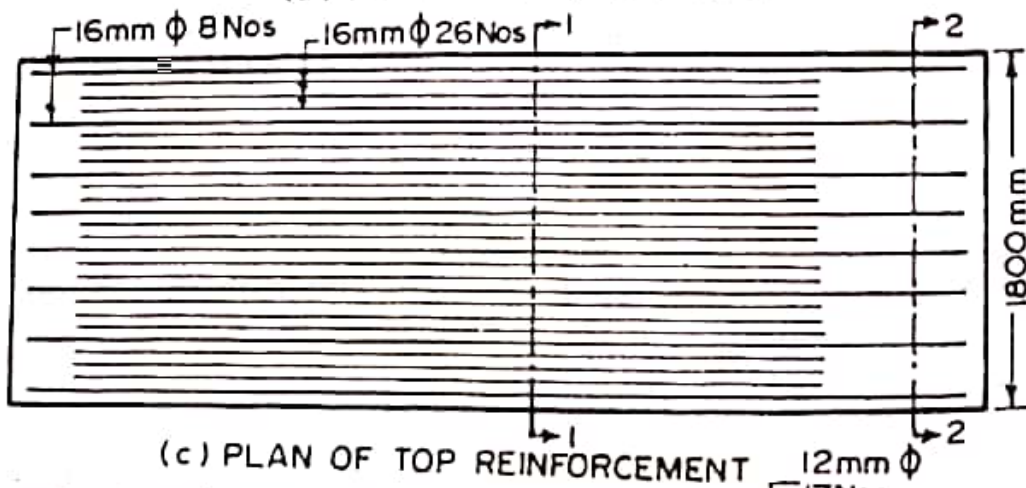
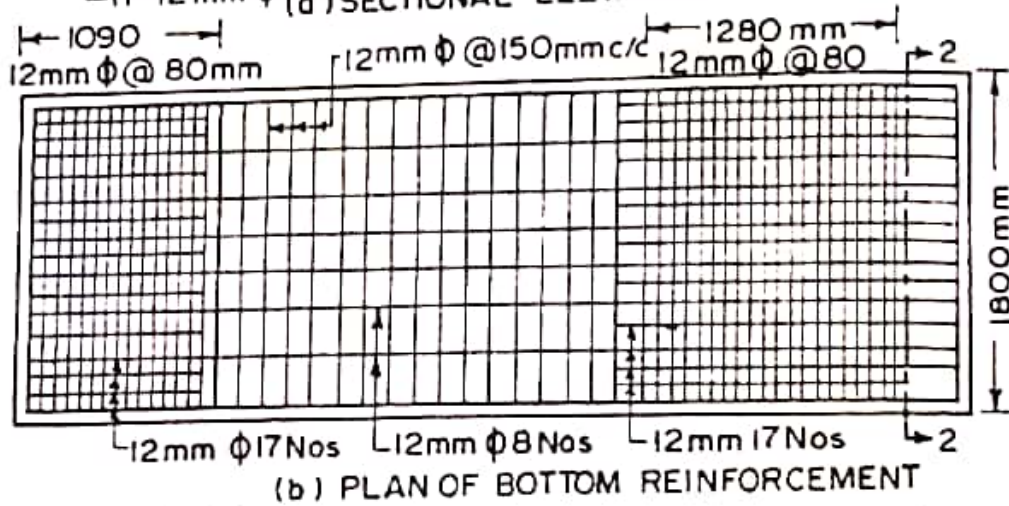
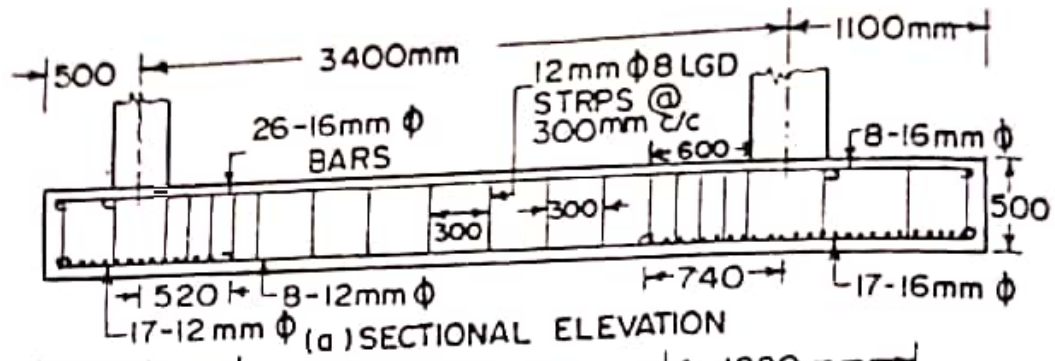
Maximum B.M. will occur the face of the column.

$$\therefore M = p_0' \frac{a^2}{2} = 254.8 \frac{(0.75)^2}{2} = 71.66 \text{ kN-m}$$

$$= 71.66 \times 10^6 \text{ N-mm}$$

$$\therefore d = \sqrt{\frac{71.66 \times 10^6}{0.874 \times 1000}} = 286 \text{ mm.}$$

Hence actual  $d = 440$  mm provided earlier is sufficient. The transverse beam will thus be of the same thickness as the rest of the footing. Since transverse reinforcement will be provided above the longitudinal one, available  $d = 440 - 12 = 428$  mm.



The transverse reinforcement is given by

$$A_{st} = \frac{71.66 \times 10^6}{140 \times 0.865 \times 428} = 1383 \text{ mm}^2$$

Using 12 mm  $\phi$  bars, spacing is given by

$$s = \frac{1000 A_{\phi}}{A_{st}} = \frac{1000 \times 113}{1383} = 82 \text{ mm}$$

Provide these @ 80 mm c/c. The reinforcement provided above should have sufficient development length.

$$L_d \approx 58.3 \phi = 58.3 \times 12 \approx 700 \text{ mm.}$$

Using 50 mm clear cover on the sides, length of bar available = 750 - 50 + anchorage value of hook. Hence safe.

Similarly, for column B, width  $B_1$  for transverse beam = 400 + 2  $\times$  440 = 1280 mm.

$$\therefore p_0' = \frac{W}{B \times B_1} = \frac{700}{1.8 \times 1.28} = 304 \text{ kN/m}^2$$

Cantilever projection  $a = \frac{1}{2} [1800 - 400] = 700 \text{ mm} = 0.7 \text{ m}$

$$\begin{aligned} \therefore M &= p_0' \frac{a^2}{2} = 304 \frac{(0.7)^2}{2} = 74.5 \text{ kN-m} \\ &= 75.5 \times 10^6 \text{ N-mm} \end{aligned}$$

$$A_{st} = \frac{74.5 \times 10^6}{140 \times 0.865 \times 428} = 1437 \text{ mm}^2$$

Using 12 mm  $\phi$  bars,  $s = \frac{1000 \times 113}{1437} \approx 80 \text{ mm.}$

Hence provide these @ 80 mm c/c in the strip of width 1280 mm. The available length of the bars will just satisfy the requirements of the development length.

For the rest of the footing, provide transverse reinforcement @ 0.15% of the area of cross-section.

$$\therefore A_{st} = \frac{0.15}{100} (1000 \times 113) = 750 \text{ mm}^2$$

$$\therefore \text{spacing of 12 mm } \phi \text{ bars} = \frac{1000 \times 113}{750} = 150 \text{ mm}$$

The details of reinforcement etc. are shown in Fig. 15.44.

**Design Example 16.6.** Fig. 16.18 shows the layout of the columns of a building. The outer columns are  $300 \times 300$  mm in size and carry a load of 500 kN each. The inner columns are  $400 \times 400$  mm in size and carry a load 800 kN each. In addition to this, each column carries a moment 160 kN-m due to wind load on the length of the building. Design a suitable raft foundation, if the bearing capacity of soil is  $100 \text{ kN/m}^2$ . Use M 15 mix. Take  $\sigma_n = 140 \text{ N/mm}^2$ .

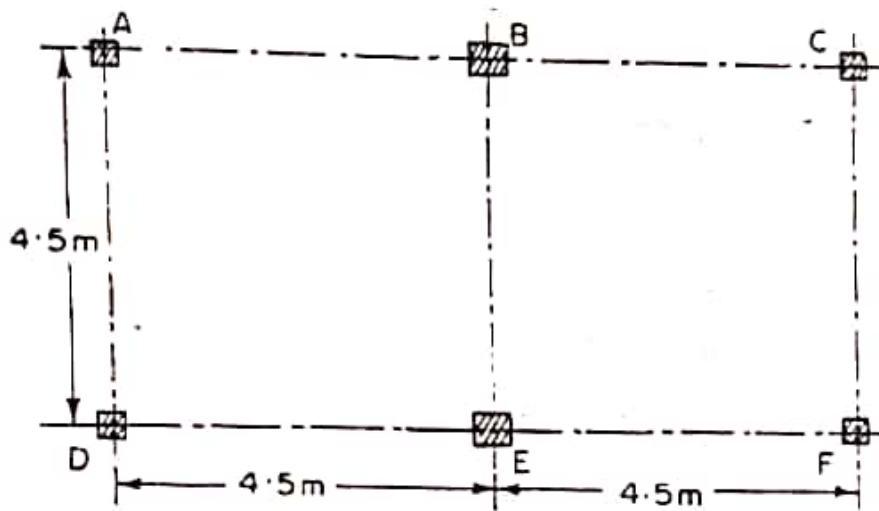


FIG. 16.18

### Solution.

#### 1. Design constants

For M 15 mix, we have the following values :  
 $\sigma_{cbc} = c = 5 \text{ N/mm}^2$  and  $\sigma_n = 140 \text{ N/mm}^2$ ;  $m = 19$ ;  
 $k = 0.404$ ;  $j = 0.865$ ;  $R = 0.874$

#### 2. Size and general arrangement of mat

$$\text{Total weight } W = (4 \times 500) + (2 \times 800) = 3600 \text{ kN}$$

$$\text{Self-weight of raft. } W' = 10\% W \text{ (say)} = 360 \text{ kN}$$

$$\therefore \text{ Total load } = 3600 + 360 = 3960 \text{ kN}$$

$$\text{Total moment } = 160 \times 6 = 960 \text{ kN-m}$$

$$\therefore \text{ Eccentricity } e = \frac{960}{3960} = 0.242 \text{ m}$$

Let the raft project 500 mm outside the column all round, so that length of raft  $= (9 + 0.3 + 2 \times 0.5) = 10.3 \text{ m}$ , and width of raft  $= (4.5 + 0.3 + 2 \times 0.5) = 5.8 \text{ m}$ .

Hence maximum and minimum intensities of pressures would be

$$= \frac{\Sigma W}{A} \left( 1 \pm \frac{6e}{B} \right) = \frac{3960}{10.3 \times 5.8} \left( 1 \pm \frac{6 \times 0.242}{5.8} \right)$$

$$\approx 83 \text{ N/m}^2 \text{ and } 5 \text{ N/m}^2$$

Since the maximum pressure is less than  $100 \text{ kN/m}^2$  the size of the footing is satisfactory.

$$\text{Net upward soil reaction } p_0 = \frac{3600}{10.3 \times 5.8} \approx 60 \text{ kN/m}^2$$

Upward reaction due to wind moment is given by

$$p_w = \pm \frac{960}{\frac{1}{8} \times 10.3 (5.8)^2} \approx \pm 16.6 \text{ kN/m}^2$$

i.e.,  $+16.6 \text{ kN/m}^2$  on leeward side and  $-16.6 \text{ kN/m}^2$  on windward side.

Pressure intensity at inner face of longitudinal beams

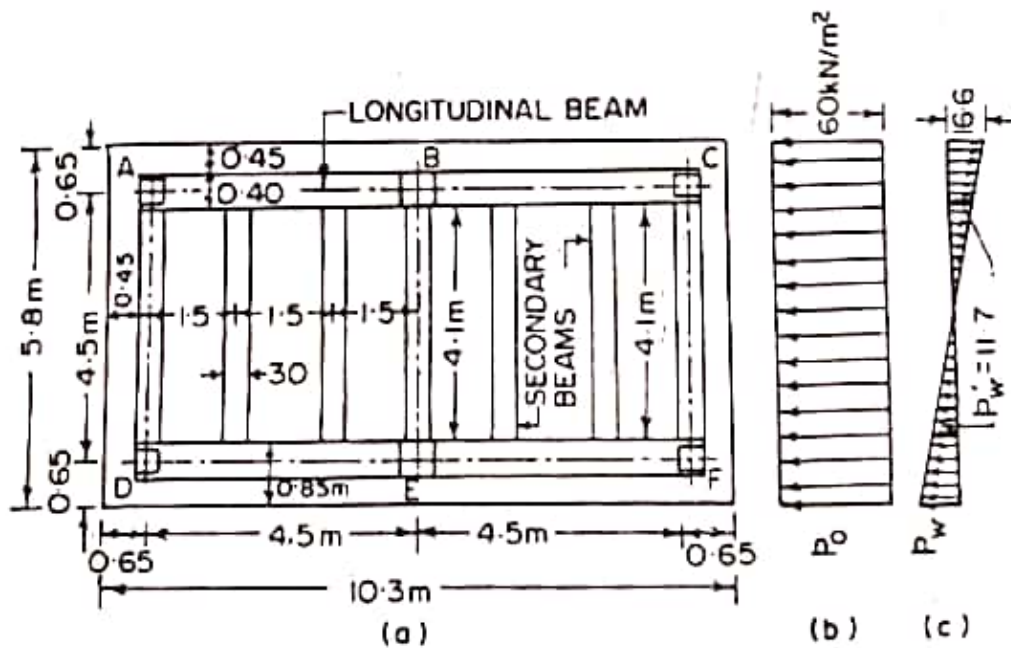


FIG. 16.19.

$$= \pm 16.6 \times \frac{4.1}{5.8} = \pm 11.7 \text{ kN/m}^2$$

Let us provide two longitudinal beams in direction  $AC$  and  $DF$ , joining three columns on each side. These two beams are joined by seven secondary beams. The general arrangement of beams etc. are shown in Fig. 16.19.

### 3. Design of slab

The soil pressure distribution due to load and wind moment are shown in Fig. 16.19 (b) and (c) respectively. Since the pressure  $p_w$  due to wind moments is less than  $\frac{1}{3}$ rd the pressure  $p_0$  due to dead load, the effect of wind will not be considered in the design of slab.

The slab is to be designed for the greater of the following two moments :

(a) Bending moment in the cantilever portion of slab.

(b) Bending moment in the continuous slab.

$$\text{Cantilever span} = 0.5 \text{ m} = a \quad \dots(i)$$

$$M_1 = p_0 \frac{a^2}{2} = \frac{60}{2} (0.5)^2 = 7.5 \text{ kN-m}$$

$$\text{shear force } F_1 = p_0 a = 60 \times 0.5 = 30 \text{ kN} \quad \dots(ii)$$

The slab is continuous over the secondary beams. Centre to centre spacing of secondary beams = 1.5 m. Taking width of secondary beams as 0.3 m, the clear span of slab, between two beams =  $1.5 - 0.3 = 1.2$  m. Hence bending moment in interior spans is

$$M_2 = \frac{p_0 L^2}{12} = \frac{60(1.2)^2}{12} = 7.2 \text{ kN-m} \quad \dots(iii)$$

$$\text{S.F. } F_2 = p_0 \frac{L}{2} = \frac{60(1.2)}{2} = 36 \text{ kN} \quad \dots(iv)$$

Hence design B.M.  $M = 7.5 \text{ kN-m} = 7.5 \times 10^6 \text{ N-mm}$

Design shear force  $F = 36 \text{ kN}$ .

The effective depth of slab is given by

$$d = \sqrt{\frac{7.5 \times 10^6}{0.874 \times 1000}} = 92.6 \text{ mm}$$

Keeping an effective cover of 50 mm and a total depth of 150 mm, available effective depth =  $150 - 50 = 100 \text{ mm}$ .

Area of reinforcement is given by

$$A_{st} = \frac{7.5 \times 10^6}{140 \times 0.875 \times 100} = 612 \text{ mm}^2$$

Using 10 mm  $\phi$  bars,  $A_{\phi} = 78.5 \text{ mm}^2$

$$\therefore \text{Spacing } s = \frac{1000 \times 78.5}{612} = 128 \text{ mm}$$

Hence provide 10 mm  $\phi$  bars @ 120 mm c/c.

Let us check the reinforcement for development length.

$$L_d \text{ required} = 58.3 \phi = 58.3 \times 10 = 583 \text{ mm.}$$

Providing a side cover of 50 mm, available length of each bar  
 $= 500 - 50 + \text{anchorage value of hook}$

$$= 450 + 13 \phi = 450 + 130 = 580 \text{ mm only.}$$

This is quite near to the required  $L_d$ . Hence O.K.

For cantilever portion, and portion under the face of the secondary beams, this reinforcement is to be provided at the bottom face of the slab since bending moment is of sagging nature, while for intermediate portion between secondary beams, the bars are to be provided on the top face, as is done in a continuous slab.

$$\begin{aligned} \text{Area of transverse reinforcement} &= \frac{0.15}{100} (1000 \times 150) \\ &= 225 \text{ mm}^2 \end{aligned}$$

$$\text{Using } 10 \text{ mm } \phi \text{ bars, spacing} = \frac{1000 \times 78.5}{225} = 348 \text{ mm.}$$

For the portion of slab projecting from the face of the main slab, cantilever projection  $= 0.45 \text{ m}$ .

$$\therefore M = \frac{60(0.45)^2}{2} = 6 \text{ kN-m}$$

Now, actual length of each bar in the cantilever portion, using 50 mm side cover  $= 450 - 50 + 13 \phi = 400 + 130 = 530 \text{ mm}$ .

$$\text{Required } L_d = 58.3 \times 10 = 583 \text{ mm.}$$

$$\text{Hence actual } \sigma_{st} \text{ carried by each bar} = \frac{530}{538} \times 140$$

$$= 127 \text{ N/mm}^2$$

$$\text{Effective depth available} = 100 - 10 = 90 \text{ mm}$$

$$\therefore A_{st} = \frac{60 \times 10^6}{127 \times 0.875 \times 90} = 6006 \text{ mm}^2$$

$$\text{and spacing of } 10 \text{ mm } \phi \text{ bars} = \frac{1000 \times 78.5}{6000} \approx 130 \text{ mm}$$

For the sake of symmetry, provide these also at 120 mm c/c. Also, the spacing of transverse reinforcement (found as 380 mm c/c) may be kept at  $2 \times 120 = 240 \text{ mm c/c}$ , so that every alternate bar of cantilever reinforcement may be used as distribution bars.



Let us check the slab for shear.

$$\tau_v = \frac{V}{bd} = \frac{F_2}{bd} = \frac{36 \times 10^3}{1000 \times 100}$$

$$= 0.36 \text{ N/mm}^2$$

$$\frac{100 A_r}{bd} = 100 \left( \frac{1000 \times 78.5}{120} \right) + (1000 \times 100) = 0.65\%$$

$\therefore$  From Table 3.1,  $\tau_c = 0.32 \text{ N/mm}^2$ .

Also, from Table 3.2,  $k = 1.3$  for  $D = 150 \text{ mm}$

$\therefore$  permissible shear stress  $= k \cdot \tau_c = 1.3 \times 0.32 = 0.42 \text{ N/mm}^2$  which is more than  $\tau_v$ . Hence safe.

#### 4. Design of intermediate secondary beam.

There are in all seven secondary beams spaced at 1.5 m c/c out which the two outer secondary beams and the central secondary beams, join columns. The remaining four secondary beams are intermediate beams which join the longitudinal beams. These intermediate beams are subjected to reaction from the slab beneath. Since the reaction due to wind moment is less than  $\frac{1}{3}$ rd the dead load reactions, it will not be considered in design.

Span of the beam = 4.5 m.

However, the load on the beam will be the upward load acting on the slab on the clear span of 4.1 m. For the remaining portion of the slab, load will be transferred directly to the longitudinal beams.

Load per metre run  $w = p_0 \times 1.5 = 60 \times 1.5 = 90 \text{ kN/m}$

The loading is shown in Fig. 16.20

End reactions transferred to each longitudinal beam

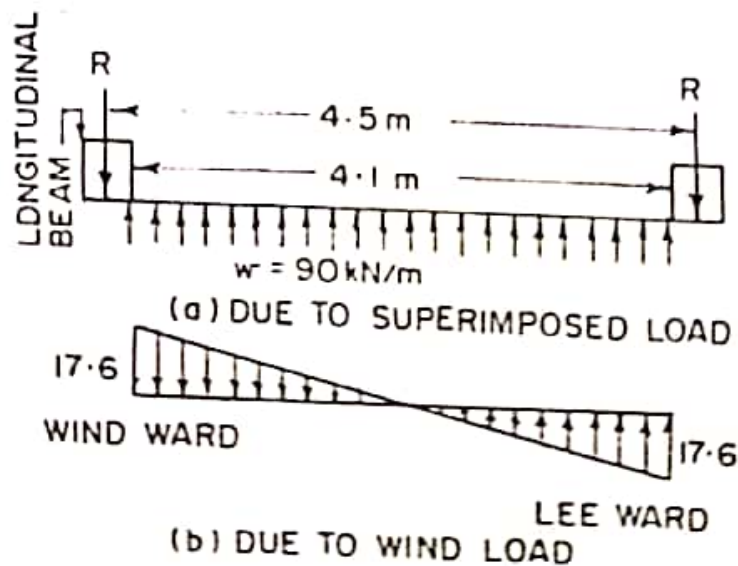


FIG. 16.20.

$$= R = \frac{1}{2} (4.1 \times 90) = 184.5 \text{ kN}$$

$$M = 184.5 \times \frac{4.5}{2} - \frac{90}{2} \left( \frac{4.1}{2} \right)^2 = 226 \text{ kN-m}$$

$$= 226 \times 10^6 \text{ N-mm}$$

$$F = R = 184.5 \text{ kN}$$

Since the beam experience hogging bending moment, it will act as T-beam. Let the width of beam  $= b_w = 300 \text{ mm}$ . The width of flange is given by

$$b_f = \frac{l_o}{6} + b_w + 6D_f$$

where  $l_o = 4.5 \text{ m} = 4500 \text{ mm}$

$$b_w = 300 \text{ mm}$$

$$D_f = 150 \text{ mm.}$$

$$\therefore b_f = \frac{4500}{6} + 300 + 6 \times 150 = 1950 \text{ mm}$$

subject to maximum of centre to centre spacing ( $= 1.5 \text{ m}$ )

Hence  $b_f = 1500 \text{ mm}$ . Taking lever arm  $= 0.9 d$ , effective depth is determined from the relation :

$$M = 0.45 b_f \sigma_{cbc} D_f \left[ \frac{2kd - D_f}{k} \right]$$

$$\therefore 226 \times 10^6 = 0.45 \times 1500 \times 5 \times 150 \left[ \frac{2 \times 0.404 d - 150}{0.404} \right]$$

$$\therefore 0.808 d = \frac{226 \times 10^6 \times 0.404}{0.45 \times 1500 \times 5 \times 150} + 150 = 330.4$$

From which  $d = 410 \text{ mm}$

$\tau_{cmax} = 1.6 \text{ N/mm}^2$  for M 15 concrete (Table 3.3)

Hence maximum depth from considerations of shear is

$$d = \frac{F}{b \cdot \tau_{cmax}}$$

or  $d = \frac{184.5 \times 10^3}{300 \times 1.6} = 384 \text{ mm.}$

Hence adopt  $d = 410 \text{ mm.}$

$$A_n = \frac{226 \times 10^6}{140 \times 0.9 \times 410} = 4375 \text{ mm}^2$$

Using 25 mm  $\Phi$  bars, No. of bars  $= \frac{4375}{491} \approx 9.$

Provide these in two layers at the upper face of the beam. Keep a vertical distance of 25 mm between them and effective cover of 60 mm. Hence total depth =  $410 + 12.5 + 12.5 + 60 = 495$ . Keep total depth of 500 mm.

Near ends, bend 3 bars downwards to take negative bending moment induced due to fixidity.

At the end, let us check for development length so as to satisfy the relation

$$\frac{M_1}{V} + L_0 \geq L_d$$

$$\text{where } M_1 = \sigma_{st} A_{st} j d = 146(6 \times 491) \times 0.9 \times 410 \\ = 1.522 \times 10^8 \text{ N-mm}$$

$$V = 184.5 \text{ kN} = 184.5 \times 10^3 \text{ N}$$

$$L_0 = \frac{l_x}{2} - x' + 13 \phi = \frac{400}{2} - 40 + 13 \times 25 = 485 \text{ mm}$$

$$L_d = 58.3 \phi = 58.3 \times 25 = 1456 \text{ mm.}$$

$$\therefore \frac{M_1}{V} + L_0 = \frac{1.522 \times 10^8}{184.5 \times 10^3} + 485 = 1310 \text{ mm.}$$

This is less than required  $L_d$ .

Hence bend only two bars downwards, so that 7 bars are available at the top face.

$$\text{Revised } M_1 = 1.522 \times 10^8 \times \frac{7}{6} = 1.78 \times 10^8$$

$$\therefore \frac{M_1}{V} + L_0 = \frac{1.78 \times 10^8}{184.5 \times 10^3} + 485 \approx 1450 \text{ mm}$$

which is quite near to  $L_d$

$$\text{Also, } \tau_v = \frac{184.5 \times 10^3}{300 \times 410} = 1.5 \text{ N/mm}^2$$

$$\frac{100 A_r}{bd} = \frac{100 \times 7 \times 491}{300 \times 410} = 2.8\%$$

Hence from Table 3.1,  $\tau_c = 0.44 \text{ N/mm}^2$

Since  $\tau_v > \tau_c$ , shear reinforcements necessary.

$$V_c = \tau_c b d = 0.44 \times 300 \times 410 = 54120 \text{ N} = 54.12 \text{ kN}$$

$$\therefore V_s = V - V_c = 184.5 - 54.12 = 130.38 \text{ kN}$$

Using 10 mm  $\phi$  4-lgd stirrups,  $A_r = 4 \frac{\pi}{4} (10)^2 = 314 \text{ mm}^2$

$$\therefore s_v = \frac{A_r \sigma_{rv} d}{V_s} = \frac{314 \times 140 \times 410}{130.38 \times 1000} = 138 \text{ mm.}$$

The distance of the point where S.F. is equal to  $V_c$  is given by  $x = \frac{184.50 - 54.12}{90} \approx 1.45$  m from the face of longitudinal beam.

Hence provide 10 mm  $\Phi$  4 lgd stirrups @ 130 mm c/c from the end to a point distant 1.45 m from the face of each longitudinal beam. For the remaining length, provide nominal stirrups at the maximum spacing given by

$$s_v = \frac{2.5 \cdot A_{sv} \cdot f_y}{b} = \frac{2.5 \times 314 \times 250}{300} = 650 \text{ mm.}$$

Subject to a maximum of 450 mm or  $0.75 d$  ( $= 0.75 \times 410 \approx 300$  mm). Hence provide 10 mm  $\Phi$  4-lgd nominal stirrups at a spacing of 300 mm c/c for the remaining length.

Let us calculate the reaction transferred by the secondary beam to main beam, due to wind load reaction  $p_r$ , since this will be useful for the design of main beam.

$$\begin{aligned} \text{Load due to } p_w, \text{ per metre run} &= \pm 1.5 p_w \\ &= \pm 1.5 \times 11.7 = \pm 17.6 \text{ kN/m} \end{aligned}$$

at the face of the main beam.

$$\begin{aligned} \text{Couple due to this} &= \left( \frac{1}{2} \times 17.6 \times 2.05 \right) \left( \frac{2}{3} \times 4.1 \right) \\ &\approx 49.3 \text{ kN-m} \end{aligned}$$

$$\therefore \text{End reactions} = \frac{49.3}{4.5} \approx 11 \text{ kN.}$$

$$\therefore \text{Net reaction at windward end} = 184.5 - 11 = 173.5 \text{ kN}$$

$$\text{Net reaction at leeward end} = 114.5 + 11 = 195.5 \text{ kN.}$$

### 5. Design of central secondary beam

The central secondary beam  $BE$  joins columns  $B$  and  $E$  and hence is subjected to end moments transmitted to it by the columns. Therefore, let us consider both  $p_o$  as well as  $p_w$ , along with the transferred end moments, to calculate the maximum bending moment and shear force. If the bending moment is greater than the intermediate secondary beam atleast by  $33\frac{1}{3}\%$ , then the design will change. Otherwise it will remain the same.

From Fig.16.19 (c), the pressure intensity  $p_w'$  due to wind, at the face of longitudinal beam = 11.7 kN/m<sup>2</sup>.

$$\therefore \text{Net pressure intensity at } E' = 60 + 11.7 = 71.7 \text{ kN/m}^2.$$

$$\text{Net pressure intensity at } B' = 60 - 11.7 = 48.3 \text{ kN/m}^2.$$

Width of slab transferring load = 1.5 m.

∴ Load intensity  $w_1$  at end  $B'$  =  $71.7 \times 1.5 = 107.6$  kN/m

Load intensity  $w_2$  at end  $B'$  =  $48.3 \times 1.5 = 72.5$  kN/m

The effective span of the beam = 4.5 m while it will receive load only in a length of 4.1 m. The loading on the beam is shown in Fig. 16.21

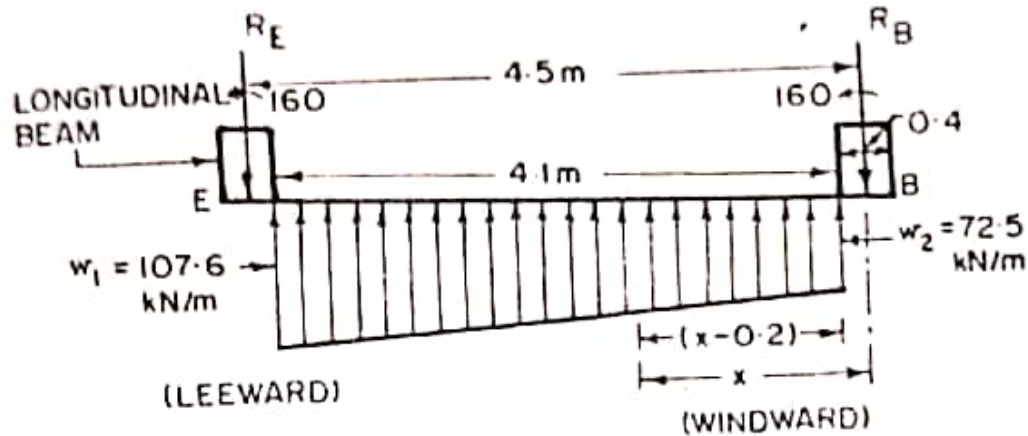


FIG. 16.21

To find reaction at  $E$ , take moments at  $B$ .

$$\begin{aligned} \therefore R_E &= \frac{1}{4.5} \left[ 72.5 \times 4.1 \left( \frac{4.1}{2} + 0.2 \right) + \frac{107.6 - 72.5}{2} \times 4.1 \right. \\ &\quad \left. \times \left\{ \frac{2}{3} (4.1) + 0.2 \right\} - 2 \times 160 \right] \\ &= 124.5 \text{ kN.} \end{aligned}$$

Similarly, taking moments  $E$ ,

$$\begin{aligned} R_B &= \frac{1}{4.5} \left[ 72.5 \times 4.1 \left( \frac{4.1}{2} + 0.2 \right) + \frac{107.7 - 72.5}{2} \times 4.1 \right. \\ &\quad \left. \times \left\{ \frac{1}{3} (4.1) + 0.2 \right\} + 2 \times 160 \right] \\ &= 244.8 \text{ kN} \end{aligned}$$

Bending moment will be maximum where S.F. is zero. Let us write down equation of shear force at  $x$  from support  $B$ .

$$F_x = 244.8 - \left[ 72.5(x - 0.2) + \frac{107.6 - 72.5}{4.1} (x - 0.2) \frac{1}{2} (x - 0.2) \right]$$

Equating this to zero, we get  $x = 3.1$  m

Hence maximum bending moment is given by

$$\begin{aligned} M &= (244.8 \times 3.1) - 160 - \left[ \frac{72.5(2.9)^2}{2} + \left( \frac{107.7 - 72.5}{4.1} \right) \right. \\ &\quad \left. \times 3.1 \times \frac{1}{2} \times 3.1 \times \frac{3.1}{3} \right] \end{aligned}$$

$$= 251.5 \text{ kN-m}$$

...(ii)

It will be seen that maximum bending moment and shear force found above by taking the wind effect into account are *less* than  $1\frac{1}{2}$  times the corresponding values for the intermediate secondary beam. Hence the same depth and same main reinforcement will be used. If however, the B.M. and S.F. found above were more than  $1\frac{1}{2}$  times the B.M. and S.F. for the intermediate secondary beam, the central beam could be designed for the above increased B.M. and S.F., taking higher permissible values of stresses.

However, due to partial fixidity at the ends, the beam will be subjected to sagging B.M.  $= \frac{WL}{24}$ , in addition to wind moment of 160 kN-m.

$$\begin{aligned} \therefore \text{Total sagging B.M.} &= (124.5 + 244.8) \frac{4.5}{24} + 160 \\ &\approx 229.2 \text{ kN-m.} \end{aligned}$$

Since the intermediate secondary beam has been designed for a B.M. of 226 kN-m, its depth  $d = 410$  mm will be sufficient to take compression, but special reinforcement at the bottom face will have to be provided at the ends. However, provide the same reinforcement as the centre, i.e. 9 bars of 25 mm  $\phi$  provided in two layers at the bottom face for the portion under the columns for a distance at least equal to  $L_d = 58.3 \times 2 \approx 1500$  mm.

#### 6. Design of end secondary beam

The end secondary beam will get less load by way of reaction from the slab, because it will receive load from slab of width  $= 0.65 + \frac{1}{2}(1.5) = 1.4$  m only against 1.5 m width for central secondary beam. However, provide the same section and reinforcement as for the central secondary beam. The ratio of load on the outer beam to the central one will be approximately equal to

$$\frac{1.4}{1.5} \approx 0.93$$

#### 7. Design of main beam

The bending moments and shear forces in the main beam are to be found under the following three conditions :

- (a) Ignoring wind effect.
- (b) Taking wind effect : leeward beam.
- (c) Taking wind effect ; windward beam.

(a) Ignoring wind effect

When there is no wind acting, the reaction transferred to the main beam by all the intermediate as well as central beams will be equal, while the reaction transferred by the end beam will be 0.93 times that transferred by intermediate beam. In addition to this, the beam will get soil reaction from slab immediately below it and the slab cantilevering from its face.

Reaction transferred by each intermediate beam = 184.5 kN

Reaction transferred by end secondary beam =  $0.93 \times 184.5$   
= 172.5 kN

Reaction transferred by slab, per metre run of beam =  $6 \times 0.85$   
=  $60 (0.4 + 0.45) = 51 \text{ kN/m}$ .

The loading diagram is shown in Fig. 16.22.

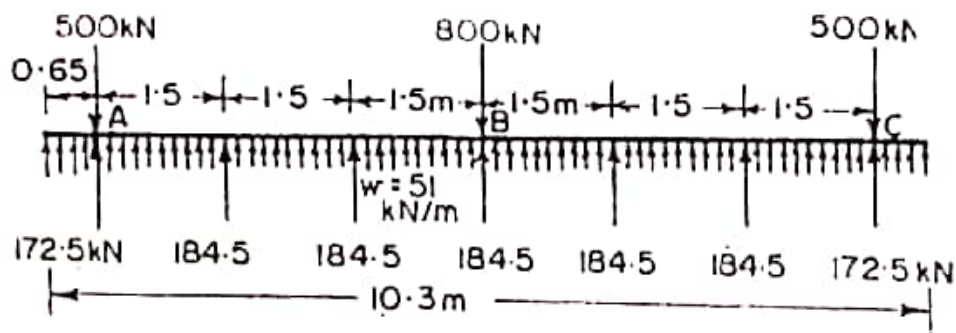


FIG. 16.22

It should be checked that total downward load of column is equal to the total upward load.

Total downward load =  $500 + 800 + 500 = 1800 \text{ kN}$

Total upward load =  $(184.5 \times 5) + (172.5 \times 2) + (10.3 \times 51)$   
= 1793

Thus, both of these are approximately equal. The discrepancy is there because of rounding off the value of  $p_0$  to  $60 \text{ kN/m}^2$  in step 2.

Maximum sagging bending moment will occur at B, its value being given by

$$M_1 = 51 \frac{(5.15)^2}{2} + (184.5 \times 1.5) + (184.5 \times 3.0) - (500 - 172.5) 4.5$$
$$= 32.8 \text{ kN-m (sagging).} \quad \dots(1)$$

Maximum hogging bending moment occurs somewhere between  $A$  and  $B$ , where shear force is zero. This is likely to occur between the two intermediate beams at distance  $x$  from  $A$ .

$$\therefore F_r = 51(x + 0.65) + 172.5 + 184.5 - 500 = 0$$

This gives  $x = 2.15$  m from  $A$  or  $2.8$  m from the edge. Hence the maximum hogging bending moment is given by

$$M_2 = (500 - 172.5) 2.15 - 184.5 (2.15 - 1.5) - \frac{51}{2} (2.8)^2$$

$$= 384.3 \text{ kN-m (hogging)} \quad \dots(2)$$

Shear force to the right of  $A$  is

$$F_1 = (500 - 172.5) - (51 \times 0.65) = 294.4 \text{ kN} \quad \dots(3)$$

Shear force to the left of  $B$  is

$$F_2 = \frac{1}{2} (800 - 184.5) = 307.8 \text{ kN}$$

(b) Wind effect : leeward beam

From step 4, reaction transferred by the intermediate secondary beam on the leeward end = 195.5 kN.

From step 5, reaction transferred by the central secondary beam on the leeward end = 124.5 kN.

Reaction transferred by the end secondary beam, on the leeward end =  $0.93 \times 195.5 - \frac{1.60 + 160}{4.5}$

$$= 182 - 71 = 111 \text{ kN}$$

Total downward load =  $500 + 800 + 500 = 1800$  kN

$\therefore$  Upward load from slab

$$= 1800 - (195.5 \times 4 + 124.5 + 111 \times 2)$$

$$= 671.50 \text{ kN}$$

Uniform upward load  $w = \frac{671.5}{10.3} = 65.2$  kN/m

The loading on the leeward beam  $DF$  is shown in Fig. 16.23

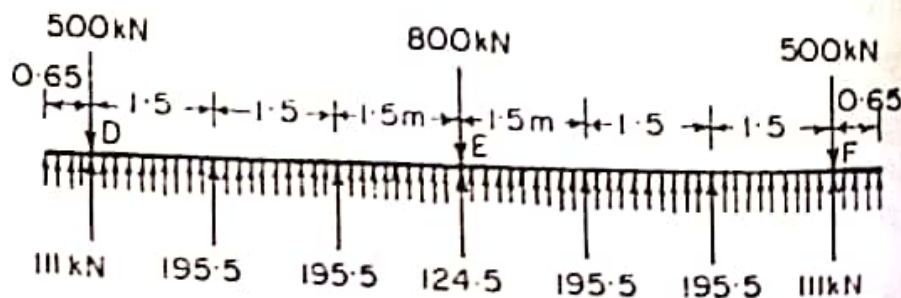


FIG. 16.23



Maximum sagging bending moment will occur at  $E$ , its value being

$$M_3 = \frac{65.2(5.15)^2}{2} + (195.2 \times 1.5) + (195.5 \times 3) - (500 - 111)4.5$$

$$= -6.1 \text{ kN-m (i.e. hogging)} \quad \dots(5)$$

Maximum hogging bending moment occurs somewhere between  $D$  and  $E$  where shear force is zero. This is likely to occur between the two intermediate beams, at distance  $x$  from  $D$ .

$$F_v = 65.2(6.5 + x) + 111 + 195.5 - 500 = 0$$

This gives  $x = 2.32$  m from  $D$ , or  $2.97$  m from the edge. Hence maximum hogging bending moment is given by

$$M_4 = (500 - 111) \times 2.32 - 195.5(2.67 - 1.5) - \frac{65.2}{3}(2.97)^2$$

$$= 327.5 \text{ kN-m (hogging)} \quad \dots(6)$$

S.F. to the right of  $D$  is

$$F_3 = (500 - 111) - (65.2 \times 0.65) = 346.6 \text{ kN} \quad \dots(7)$$

S.F. to the left of  $E$  is

$$F_4 = \frac{1}{2}(800 - 124.5) = 337.8 \text{ kN} \quad \dots(8)$$

**(c) Wind effect : windward beam**

From step 4, reaction transferred by the intermediate secondary beam on the windward side = 173.5 kN

From step 5, reaction transferred by central secondary beam on the windward end = 244.8 kN.

Reaction transferred by the end secondary beam on the windward end =  $(0.93 \times 173.5) + \frac{160 + 160}{4.5} = 232.5$  kN

Total downward load =  $500 + 800 + 500 = 1800$  kN

$\therefore$  Upward load from the slab =  $1800 - (4 \times 173.5 + 248.8 + 2 \times 232.5) = 396.2$  kN.

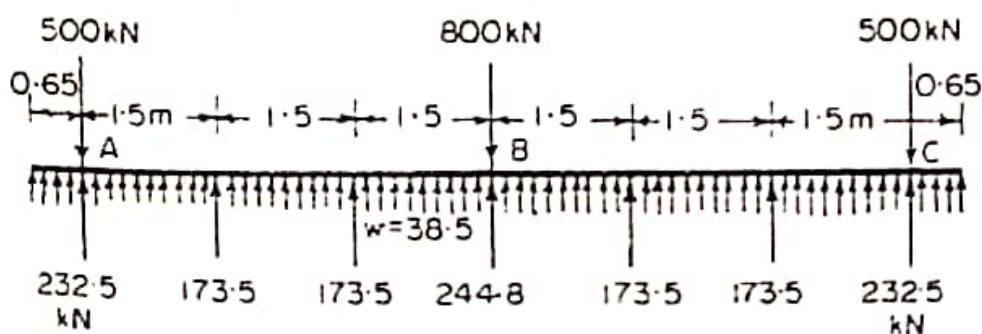


FIG. 16.24.

$$\therefore \text{Upward } w = \frac{396.2}{10.3} \approx 38.5 \text{ kN/m.}$$

The loading on the windward main beam  $AC$  is shown in Fig 16.24.

Maximum sagging bending moment will occur at  $B$  :

$$\begin{aligned} M_s &= \frac{38.5(5.15)^2}{2} + (2 \times 173.5 \times 2.25) - (500 - 232.5) \times 4.5 \\ &= 87.6 \text{ kN-m} \end{aligned}$$

Maximum hogging bending moment will occur where S.F. is zero, some where between  $A$  and  $B$ . Let it occur at  $x$  from  $A$  between the two intermediate beams.

$$F_x = 38.5(x + 0.65) + 173.5 + 232.5 - 500 = 0$$

where gives  $x = 1.79$  m from  $A$  or 2.44 m from the edge.

Hence maximum hogging bending moment is given by

$$\begin{aligned} M_h &= (500 - 232.5) \times 1.79 - 173.5(1.79 - 1.5) - \frac{38.5}{2}(2.44)^2 \\ &= 314 \text{ kN-m (hogging)} \end{aligned} \quad \dots(10)$$

S.F. to the right of  $A$  is given by

$$F_s = (500 - 232.5) - (0.65 \times 38.5) = 242.5 \text{ kN} \quad \dots(11)$$

S.F. to the left of  $B$  is

$$F_b = \frac{1}{2}(800 - 244.8) = 277.6 \text{ kN} \quad \dots(12)$$

#### *Design values*

From the above cases, we get the following values for design purposes :

Without wind : Maximum sagging moment = 32.8 kN-m

Maximum hogging moment = 384.3 kN-m

Maximum shear = 307.8 kN

With wind : Maximum sagging moment = 87.6 kN-m

Maximum hogging moment = 327.5 kN-m

Maximum shear = 346.6 kN

Thus critical conditions are under 'no wind'. T-beam action will be available for maximum hogging bending moment = 384.3 kN-m. Let the width of rib = 400 mm.

The flange width of T-beam is given by

$$b_f = \frac{l_0}{6} + b_w + 6 D_f$$

where  $l_0$  = distance between points of zero moments in the beam

$$= 10.3 - (2 \times 2.8) = 4.7 \text{ m} = 4700 \text{ mm}$$

$$b_w = 400 \text{ mm}$$

$$D_f = 150 \text{ mm}$$

$$\therefore b_f = \frac{4700}{6} + 400 + 6 \times 150 \approx 2083 \text{ mm.}$$

However, since the beam is situated nearly at the outer boundary, its half flange is restricted in its width to a value  $= 0.20 + 0.45 = 0.65 \text{ m} = 650 \text{ mm}$ . Hence maximum value of  $b_f$  is restricted to  $2 \times 650 = 1300 \text{ mm}$ .

Hence adopt  $b_f = 1300 \text{ mm}$ . Taking lever arm  $= 0.9 d$ , the effective depth is given by the relation :

$$M = 0.45 b_f \sigma_{cbc} D_f \left( \frac{2kd - D_f}{k} \right)$$

$$\text{or } 384.3 \times 10^6 = 0.45 \times 1300 \times 5 \times 150 \left( \frac{2 \times 0.404 d - 150}{0.404} \right)$$

$$\text{or } 0.808 d = \frac{384.3 \times 10^6 \times 0.404}{0.45 \times 1300 \times 5 \times 150} + 150$$

From which  $d = 624 \text{ mm}$ .

This depth will be sufficient to take the sagging bending moment when the beam will act as a simple rectangular beam.

$$A_{st} = \frac{384.3 \times 10^6}{140 \times 0.9 \times 624} = 4888 \text{ mm}^2$$

$$\text{Using } 25 \text{ mm } \phi \text{ bars, No. of bars} = \frac{4888}{490} \approx 10.$$

Hence provide these at the top face of the beam in two layers. This reinforcement should be safe to satisfy the development length criterion at the point of contraflexure, so as to satisfy the relation

$$\frac{M}{V} + L_0 \geq L_d.$$

Assuming that all the bars are continued even beyond the point of contraflexure.

$$M = \sigma_{st} A_{st} j d = 140 (10 \times 490) \times 0.9 \times 624 \\ = 3.85 \times 10^8 \text{ N-mm}$$

$V$  = Shear at the point of contraflexure

Assuming that the point of contraflexure falls just to the left of B, shear force  $V = F_2 = 307.8 \text{ kN}$

$$L_0 = 12 \phi \text{ or } d \text{ whichever is more} = 624 \text{ mm}$$

$$L_d = 58.3 \times 25 = 1458 \text{ mm.}$$

$$\therefore \frac{M}{V} + L_n = \frac{3.85 \times 10^8}{30.78 \times 10^1} + 624 = 1875 \text{ mm.}$$

Hence safe

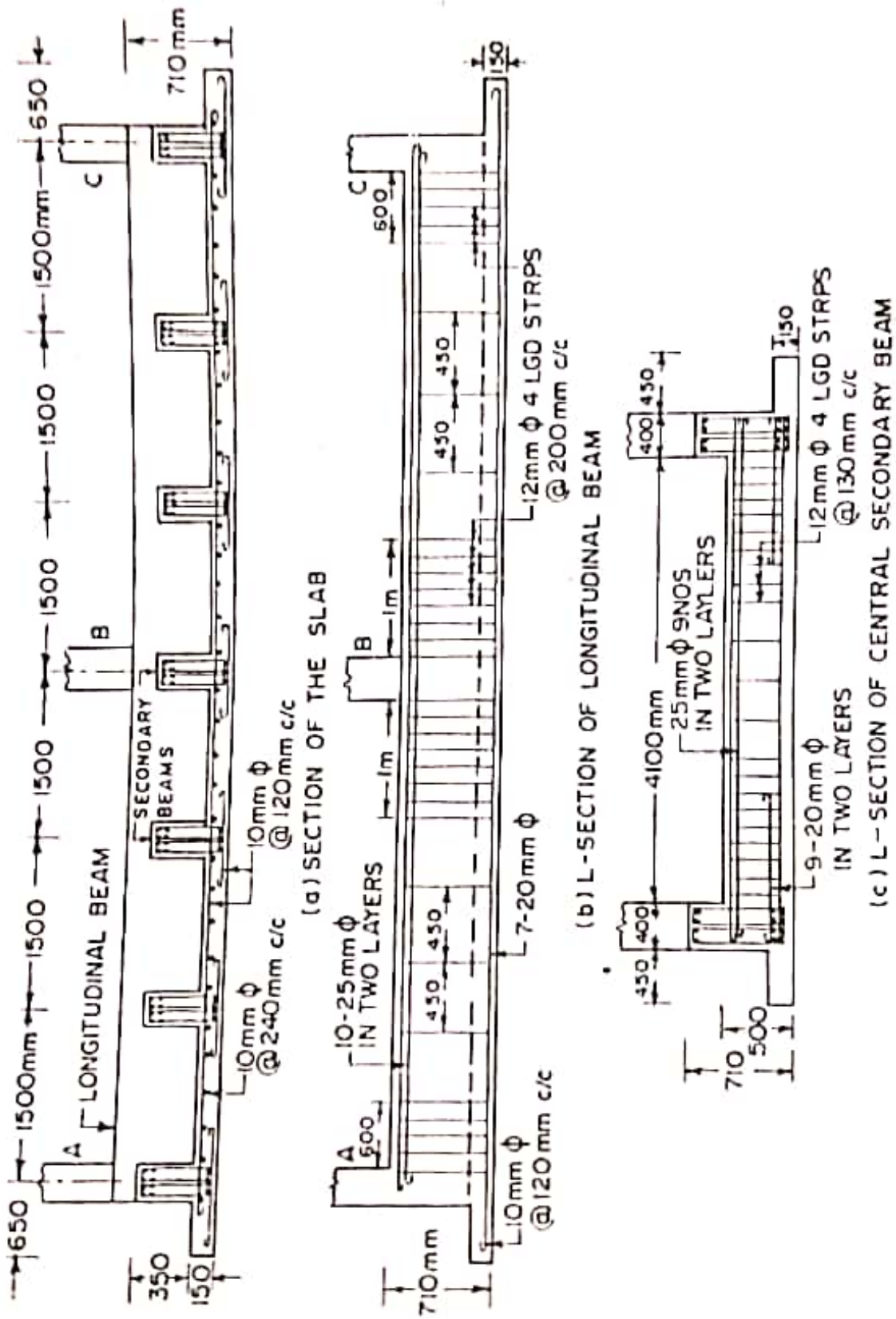


FIG. 16.25

Continue all the 10 bars throughout the length, in two layers.  
Total depth of beam =  $624 + 24 + 60 \approx 710$  mm.

The beam should also have reinforcement at its bottom face to resist sagging bending moment at the centre and at the ends, for a bending moment of 87.6 kN-m under wind load conditions. The reinforcement required for bending moment will be less than 2 bars of 25 mm  $\phi$ . However, this reinforcement should satisfy the criterion of development length at the point of contraflexure near the centre of beam, where S.F. is maximum.

Let us use  $n$  number of 20 mm dia. bars each having  $A_{\phi} = 314$  mm<sup>2</sup>.

$$\begin{aligned} \therefore M &= \sigma_{st} A_{st} j d = 140 (314 n) \times 0.9 \times 624 \\ &= n \times 24.69 \times 10^6 \text{ N-mm} \end{aligned}$$

$$\begin{aligned} V &= \text{S.F. at the point of contraflexure} \\ &\approx 307.8 \text{ kN (assuming this to be equal to S.F. at B)} \end{aligned}$$

$$L_0 = 12 \phi \text{ or } d, \text{ whichever is more} = 624 \text{ mm.}$$

$$L_d = 58.3 \phi = 58.3 \times 20 = 1166 \text{ mm.}$$

$$\begin{aligned} \therefore \frac{M}{V} + L_0 &= \frac{n \times 24.69 \times 10^6}{307.8 \times 10^3} + 624 \\ &= (80.2 n + 624) \end{aligned}$$

Equating this to  $L_d$ ,

$$80.2 n + 624 = 1166$$

From which  $n = 6.76 \approx 7$ .

Hence provide 7 bars of 20 mm diameter at the bottom face, throughout the length of the beam.

Let us now check the beam for shear force of 307.8 kN.

$$\tau_v = \frac{307.8 \times 1000}{400 \times 624} = 1.23 \text{ N/mm}^2 \text{ (at B).}$$

$$\frac{100 A_s}{bd} = \frac{100(10 \times 490)}{400 \times 624} = 1.96\%$$

Hence  $\tau_c = 0.44$  N/mm<sup>2</sup> (Table 3.1)

Since  $\tau_v > \tau_c$ , shear reinforcement is necessary.

$$V_c = 0.44 \times 400 \times 624 = 109824 \text{ N} \approx 109.8 \text{ kN}$$

$$\therefore V_s = 307.8 - 109.8 = 198 \text{ kN.}$$

Using 12 mm  $\phi$  4-lgd stirrups,  $A_{sv} = 452$  mm<sup>2</sup>.

$$\therefore \text{Spacing} = \frac{\sigma_{sv} A_{sv} d}{V_s} = \frac{140 \times 452 \times 624}{198 \times 1000} \approx 200 \text{ mm}$$

Hence provide the stirrups @ 200 mm c/c. for 1 m on either side of the faces of the column B.

Similarly, shear force at  $A = 294.4$  kN. Hence provide 12 mm  $\Phi$  4-legged stirrups @ 200 mm c/c, for a distance of 0.6 m to the right of face of column  $A$ , and 0.6 m to left of column  $C$ .

For the remaining length, provide nominal shear reinforcement at a maximum spacing of

$$s_v = \frac{2.5 A_{sv} f_y}{b} = \frac{2.5 \times 452 \times 250}{400}$$
$$\approx 700 \text{ mm.}$$

But not more than 450 mm nor more than  $0.75 d$  ( $= 0.75 \times 624 = 468$  mm).

Hence provide 12 mm  $\Phi$  4-lgd nominal stirrups @ 450 mm c/c for the remaining length. The details of reinforcement etc. are shown in Fig. 16.25.

# Pile Foundations

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## 17.1 TYPES OF PILES.

The use of piles as foundation can be traced since olden times. The art of driving piles was well established in Roman times and the details of such foundations were recorded by Vitruvius in 59 A.D. Today, pile foundation is much more common than any other type of deep foundation. Based on foundation or the use, piles may be classified as : (1) end bearing pile (2) friction pile (3) compaction pile (4) tension pile or uplift pile (5) anchor pile (6) fender pile and dolphins (7) batter pile (8) sheet pile.

*End bearing piles* are used to transfer load through water or soft soil to a suitable bearing stratum. *Friction piles* are used to transfer loads to a depth of a friction load-carrying material by means of skin friction along the length of piles. *Compaction piles* are used to compact loose granular soils, thus increasing their bearing capacity. *Tension or uplift piles* anchor down the structures subjected to uplift due to hydrostatic pressure or due to overturning moment. *Anchor piles* provide anchorage against horizontal pull from sheet piling or other pulling forces. *Fender piles and dolphins* are used to protect water front structures against impact from ships or other floating objects. *Sheet piles* are commonly used as bulk heads or as impervious cut off to reduce seepage and uplift under hydraulic structures. *The batter piles* are used to resist large horizontal or inclined forces.

Piles can also be classified, based on materials composition, as follows :

1. *Concrete piles*
  - (a) Precast.
  - (b) Cast-in-situ :
    - (i) driven piles ; cased or uncased.

(ii) bored piles ; pressure piles and under reamed pile.

2. *Timber piles*

3. *Steel piles*

(a) H-pile.

(b) Pipe pile.

(c) Sheet pile.

4. *Composite piles*

(a) Concrete and timber.

(b) Concrete and steel.

The *precast concrete piles* are generally used for a maximum design load of about 800 kN except for large prestressed piles. They must be reinforced to withstand handling stresses. They require space for casting and storage, more time to set and cure before installation and heavy equipment for handling and driving. They also incur large cost in cutting off extra length or adding more length. The *cast-in-situ* piles are generally used for a maximum design load of 750 kN except for compacted, pedestal piles. They are installed by pre-excavation, thus eliminating vibration due to driving and the handling stresses. Cast-in-place piles may be classified into two classes : driven piles, (cased or uncased) and bored piles (pressure piles, pedestal piles and under-reamed piles). A variety of cast-in-situ piles are in use, each bearing the name of the manufacturer. The common types are as follows : (i) Raymond standard pile (ii) Raymond step-taper- pile (iii) Union metal pile or monotub, (iv) MacArthur compressed uncased pile (v) Mac Arthur cased pile. (vi) Franki standard pile. (vii) Western button bottom pile etc.

*Under reamed pile* is a special type of bored pile having an increased diameter or bulb at some point in its length, to anchor the foundation in expansive soil subjected to alternative expansion and contraction.

*Concrete filled steel pipes* and steel *H-piles* are used as long piles with high bearing capacity. They are rarely used unless they reach a stratum of exceptionally high supporting capacity, since their cost is very high. *Timber piles* have small bearing capacity and are not permanent unless treated. They are prone to damage by hard driving, and should not be driven through hard stratum or boulders. *Composite piles* are suitable where the upper part of the pile is to project above the water table. Such a pile consists of a lower portion of untreated timber and an upper portion of concrete. In the other type of composite pile, steel piles are attached to their lower end of cast in place concrete piles. This type is used in cases where the required length of pile is greater than that available for the cast-in-place type.



**Design Example 17.1.** Design a pile under a column transmitting an axial load of 600 kN. The pile is to be driven to a hard stratum available at a depth of 8 metres. Take  $\sigma_{cc} = 4 \text{ N/mm}^2$  and  $\sigma_{sc} = 130 \text{ N/mm}^2$ .

**Solution.**

**1. Main reinforcement**

Let the length of the pile above ground, including pile cap, etc. = 0.6 m.

$\therefore$  Total length of pile = 8.6 m.

Let the size of the pile be 400 mm  $\times$  400 mm

$$\frac{l}{D} \text{ ratio} = \frac{8.6}{0.4} = 21.5$$

Since this is greater than 12, the pile behaves as long column.

Hence reduction coefficient  $C_r = 1.25 - \frac{l_f}{48 D}$

$$= 1.25 - \frac{8.6}{48 \times 0.4} \approx 0.8$$

$\therefore$  Design load for a short column =  $\frac{600}{0.8} = 750 \text{ kN}$

Load carrying capacity of column is given by

$$P = \sigma_{cc} A_c + \sigma_{sc} A_{sc}$$

where  $A_c =$  area of concrete =  $(400 \times 400) - A_{sc} = 16 \times 10^4 - A_{sc}$

$$\therefore 750 \times 10^3 = 4 (16 \times 10^4 - A_{sc}) + 130 A_{sc}$$

From which  $A_{sc} = 873 \text{ mm}^2$ .

Since the length of pile is less than 30 times the width, minimum reinforcement @ 1.25% of gross cross-sectional area  $= \frac{125}{100} (400 \times 400) = 2000 \text{ mm}^2$ .

However, provide 4 bars of 25 mm  $\phi$  giving total area of steel  $= 4 \times 490 = 1960 \text{ mm}^2$  which is quite close to the minimum requirements. Provide an effective cover of 40 mm.

### 2. Lateral reinforcement in the body of the pile

Lateral reinforcement in the body of pile is provided @ 0.2% of gross volume.

$$\begin{aligned} \therefore \text{Volume needed per mm length} &= \frac{0.2}{100} (400 \times 400 \times 1) \\ &= 320 \text{ mm}^3. \end{aligned}$$

$$\text{Clear cover to main reinforcement} = 40 - 12.5 = 27.5 \text{ mm}$$

Using 8 mm  $\phi$  ties, length of each side of tie  $= 400 - 2 \times 27.5 + 8 = 353 \text{ mm}$

$$\text{Area } A_{\phi} = \frac{\pi}{4} (8)^2 = 50.3 \text{ mm}^2.$$

$$\begin{aligned} \therefore \text{Volume of each tie} &= 4 \times 353 \times 50.3 \\ &= 70975 \text{ mm}^3 \end{aligned}$$

$$\therefore \text{Pitch} = \frac{70955}{320} = 222 \text{ mm}$$

Maximum pitch permissible

$$= \frac{1}{2} \times 400 = 200 \text{ mm}.$$

Hence provide 8 mm  $\phi$  ties @ 200 mm c/c throughout the length of the pile.

### 3. Lateral reinforcement near pile head

Near pile head, special spiral reinforcement is to be provided for a length of  $3 \times 400 = 1200 \text{ mm}$ . Volume of spiral, @ 0.6% of gross volume, per mm length is

$$= \frac{0.6}{100} (400 \times 400 \times 1) = 960 \text{ mm}^3$$

Using 8 mm  $\phi$  spiral, having  $A_{\phi} = 50.3 \text{ mm}^2$ , pitch is given by

$$s = \frac{\text{circumference of spiral} \times A_{\phi}}{960}$$

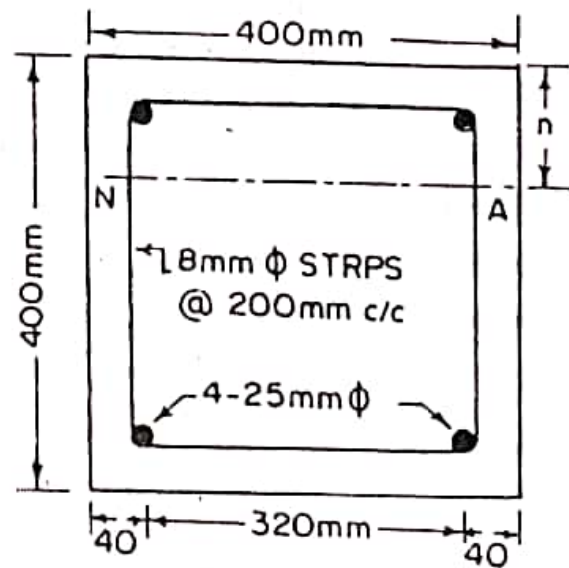


FIG. 17.4

$$= \frac{\pi \times 353 \times 50.3}{960} = 58 \text{ mm}$$

Provide the spiral at 55 mm pitch. Provide 6 additional bars of 16 mm  $\phi$  vertically within the spiral. These spirals will be in addition to the normal ties.

#### 4. Lateral reinforcement near pile end

Volume of ties per mm length @ 0.6% of gross volume = 960 mm<sup>3</sup>.

$$\therefore \text{Volume of each tie} = 70975 \text{ mm}^3.$$

$$\therefore \text{Pitch} = \frac{70975}{960} = 74 \text{ mm.}$$

Provide ties @ 70 mm c/c in a bottom length of  $3 \times 400 = 1200 \text{ mm}$ .

#### 5. Spacer forks and links

Provide two pairs of 12 mm  $\phi$  spacer fork with 6mm  $\phi$  links @ 1.5 m c/c along the length.

#### 6. Check for handling stresses

Provide three holes in the pile as follows :

(i) One hole at  $0.293L = 0.293 \times 8.6 \approx 2.5 \text{ m}$  from the pile for the purpose of hoisting it.

(ii) One hole each from either end, at a distance of  $0.206L = 0.206 \times 8.6 \approx 1.75 \text{ m}$  for the purpose of stacking.

$$\begin{aligned} \text{(iii) Weight of pile per meter run} &= 0.4 \times 0.4 \times 1 \times 25000 \\ &= 4000 \text{ N/m} \end{aligned}$$

$$\therefore M = \frac{4000 (2.5)^2}{2} = 12500 \text{ N-m} = 125 \times 10^5 \text{ N-mm}$$

Effective depth of pile section =  $400 - 40 = 360 \text{ mm}$ . Let the neutral axis be situated at  $n$  below one face. Equating the moment of area about the N.A. we get (Fig. 17.4).

$$\frac{b n^2}{2} + (m_c - 1) A_{sc} (n - d_c) = m A_{st} (d - n)$$

$$\text{or } \frac{400}{2} n^2 + (1.5 \times 19 - 1) 980 (n - 40) = 19 \times 980 (360 - n)$$

$$\text{or } n^2 + 227.8 n - 38900 = 0$$

which gives  $n = 113 \text{ mm}$ .

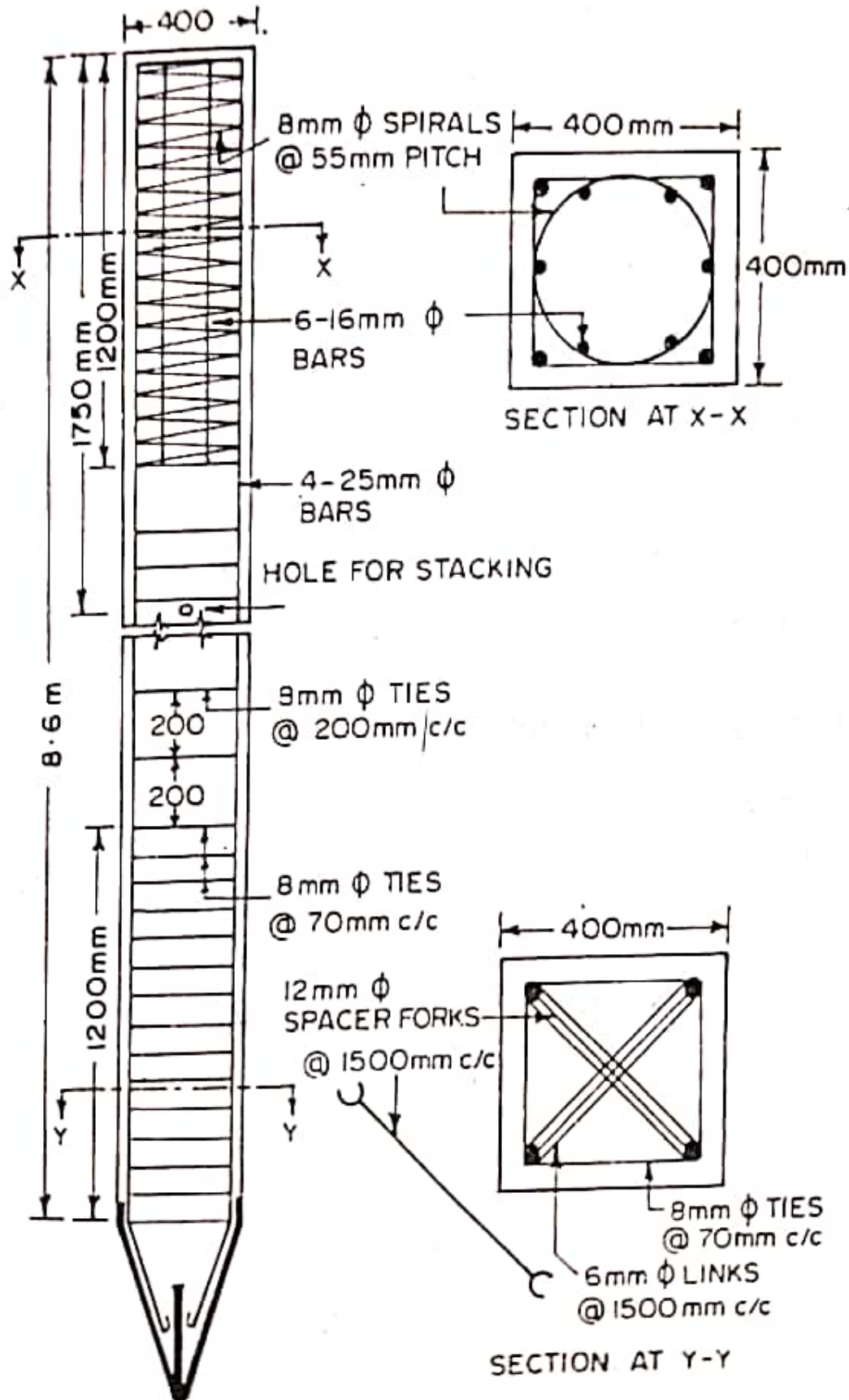
Taking moment of forces about tensile steel, we get

$$b n \frac{c}{2} \left[ d - \frac{n}{3} \right] + (1.5 m - 1) A_{sc} c' (d - d_c) = M$$

$$\text{where } c' = \frac{(n - d_c) c}{n} = \frac{113 - 40}{113} \times c = 0.65 c$$

$$\therefore 400 \times 113 \frac{c}{2} \left[ 360 - \frac{113}{3} \right] + 27.5 \times 980 \times 0.65 c (360 - 40)$$

$$= 125 \times 10^5$$



# Retaining Walls

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## 18.1. INTRODUCTION

A *retaining wall* or retaining structure is used for maintaining the ground surfaces at different elevations on either side of it. Whenever embankments are involved in construction, retaining walls are usually necessary. In the construction, of buildings having basements, retaining walls are mandatory. Similarly in bridge work, the wing walls and abutments etc. are designed as retaining walls, to resist earth pressure along with superimposed loads. The material retained or supported by a retaining wall is called *backfill* which may have its top surface horizontal or inclined. The position of the backfill lying above the horizontal plane at the elevation of the top of a wall is called the *surcharge*, and its inclination to the horizontal is called the *surcharge angle*  $\beta$ .

In the design of retaining walls or other retaining structures, it is necessary to compute the lateral earth pressure exerted by the retaining mass of soil. The equation of finding out the lateral earth pressure against retaining wall is one of the oldest in the Civil Engineering field. The *plastic state of stress*, when the failure is imminent, was investigated by Rankine in 1860. A lot of theoretical and experimental work has been done in this field and many theories and hypothesis have been proposed.\*

## 18.2. TYPES OF RETAINING WALLS

Retaining walls may be classified according to their mode of resisting the earth pressure, and according to their shape. Following are some of common types of retaining walls (Fig. 18.1).

- (i) Gravity walls
- (ii) Cantilever retaining walls
  - (a) T-shaped
  - (b) L-shaped
- (iii) Counterfort retaining walls.
- (iv) Buttressed walls.

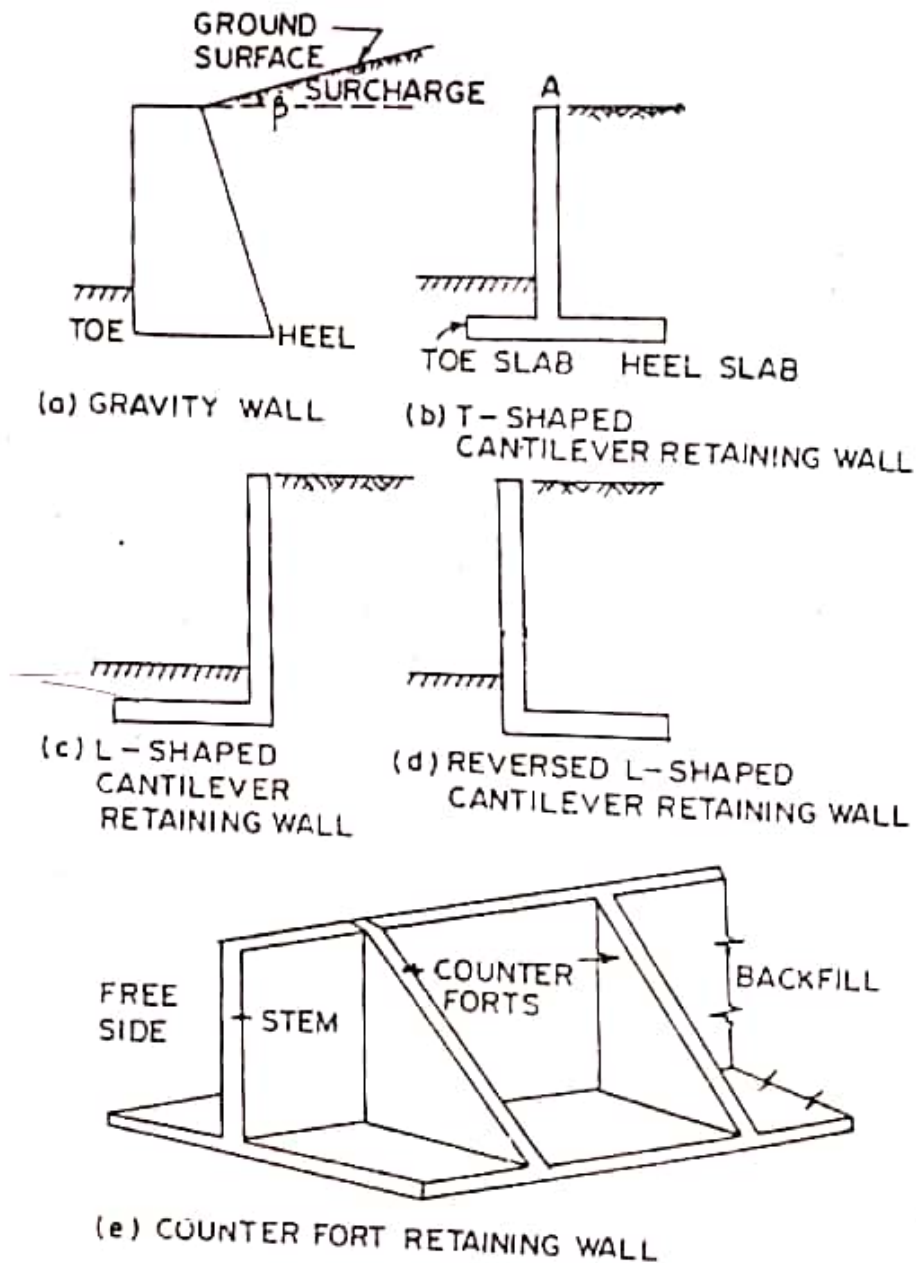


FIG. 18.1. VARIOUS TYPES OF RETAINING WALLS.

A gravity retaining wall shown in Fig. 18.1(a) is the one in which the earth pressure exerted by the backfill is resisted by dead weight of the wall, which is either made of masonry or of mass

concrete. The stress developed in the wall is very low. These walls are so proportioned that no tension is developed anywhere, and the resultant of forces remain within the middle third of the base.

The cantilever retaining wall resists the horizontal earth pressure as well as other vertical pressures by way of bending of various components acting as cantilevers. A common form of cantilever retaining wall is the T-shaped wall shown in Fig. 18.1(b). The wall consists of stem  $AB$ , heel slab  $BC$  and toe slab  $DB$ . Each of these bend as cantilevers, about  $B$ . They are, therefore, reinforced on the tension face. Another form of cantilever retaining walls are the L-shaped walls shown in Fig. 18.1(c) and (d). They also resist the soil pressures by bending.

A counterfort retaining wall is shown in Fig. 18.1(e). The vertical stem and the heel slab are strengthened by providing counterforts at some suitable intervals. Because of provision of counterforts, the vertical stem as well as the heel slab acts as *continuous slab*, in contrast to the cantilevers of cantilever retaining wall. The toe slab however acts as cantilever bending upwards. This type of retaining wall is used when backfill of greater height is to be retained. A buttressed wall is a modification of the counterfort retaining wall in which the counterforts, called the buttresses, are provided to the other side of the backfill. However the buttresses reduce the clearance in front of the wall, and therefore these walls are commonly used.

## 18.7. DESIGN OF CANTILEVER RETAINING WALL WITH HORIZONTAL BACKFILL

**Design Example 18.1.** Design a T-shaped cantilever retaining wall to retain earth embankment 3 m high above ground level. The unit weight of earth is  $18 \text{ kN/m}^3$  and its angle of repose is  $30^\circ$ . The embankment is horizontal at its top. The safe bearing capacity of soil may be taken as  $100 \text{ kN/m}^2$  and the coefficient of friction between soil and concrete as 0.5. Use M 15 mix. Take  $\sigma_n = 140 \text{ N/mm}^2$ .

### Solution

#### 1. Design constants

For M 15 concrete and mild steel reinforcement we have the following values :

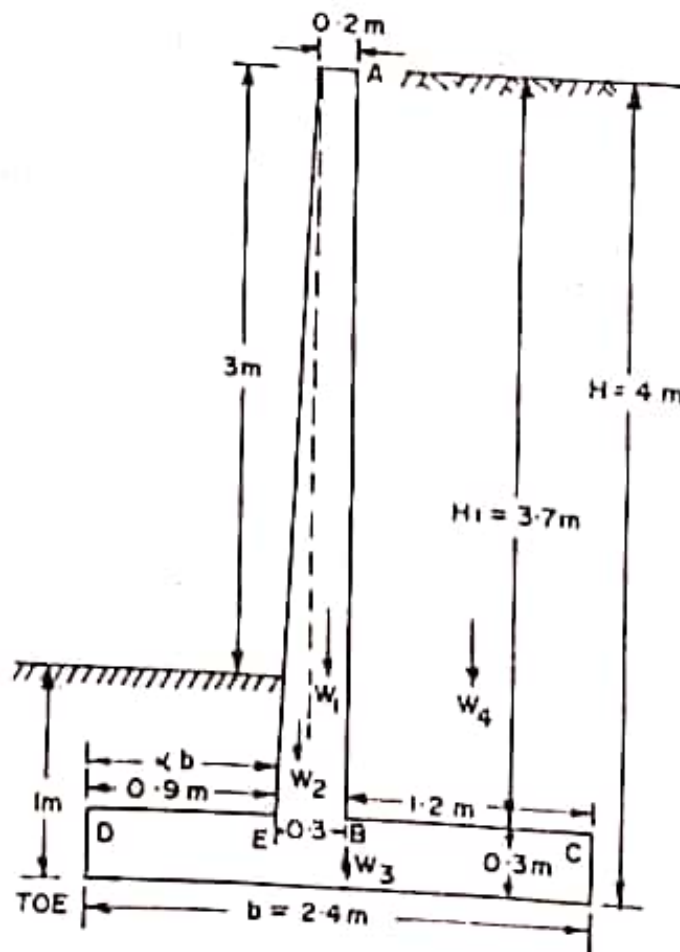
$$c = \sigma_{cbc} = 5 \text{ N/mm}^2 ; t = \sigma_n = 140 \text{ N/mm}^2 ; m = 19$$

$$k = 0.404 ; j = 0.865 \text{ and } R = 0.874$$

#### 2. Depth of foundation

$$\gamma = 18 \text{ kN/m}^3 = 18000 \text{ N/m}^3$$

The minimum depth of foundation is





$$\begin{aligned}
 y_{min} &= \frac{q_c}{\gamma} \left( \frac{1 - \sin\phi}{1 + \sin\phi} \right)^2 \\
 &= \frac{100}{18} \left( \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} \right)^2 \\
 &= 0.62 \text{ m.}
 \end{aligned}$$

However, keep depth = 1 m to accommodate thickness of base wall below the ground surface.

Hence height of wall above its base =  $H = 3 + 1 = 4$  m.

### 3. Dimensions of base

The ratio of the length of toe slab ( $DE$ ) to the base width  $b$  is given by Eq. 18.26 :

$$\alpha = 1 - \frac{q_c}{2.2\gamma H} = 1 - \frac{100}{2.2 \times 18 \times 4} = 0.365$$

Keep  $\alpha = 0.37$

...(i)

The width of base is given by Eq. 18.25.

$$b = 0.95 H \sqrt{\frac{K_a}{(1 - \alpha)(1 + 3\alpha)}}$$

$$k_a = \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = \frac{1}{3}$$

$$b = 0.95 \times 4 \sqrt{\frac{1}{3(1 - 0.37)(1 + 3 \times 0.37)}} = 1.90 \text{ m.}$$

The base width from the considerations of sliding is given by Eq. 18.27.

$$b = \frac{0.7H K_a}{(1 - \alpha)\mu} = \frac{0.7 \times 4}{1 - 0.37} \times \frac{1}{3 \times 0.5} = 2.96 \text{ m.}$$

This width is excessive. Normal practice is to provide  $b$  between 0.4 to 0.6  $H$ . Taking maximum value of 0.6  $H$ ,

$$b = 0.6 H = 0.6 \times 4 = 2.4 \text{ m.}$$

Hence provide  $b = 2.4$  m. The wall will be unsafe against sliding. This will be made safe by providing a shear key at the base.

Width of toe slab =  $0.37 \times 2.4 = 0.89$  m.

Provide toe slab 0.9 m long.

Let the thickness of base be  $\frac{1}{12}H \approx 0.3$  m for preliminary calculations.

### 4. Thickness of stem

Height  $AB = 4 - 0.3 = 3.7$  m.

Consider one meter length of retaining wall.

Maximum bending moment at  $B$

$$= K_a \gamma \frac{H_1^3}{6} = \frac{1}{3} \frac{18}{6} (3.7)^3$$

$$= 50.65 \text{ kN-m} = 50.65 \times 10^6 \text{ N-mm}$$

Hence the effective depth is

$$d = \sqrt{\frac{50.65 \times 10^6}{1000 \times 0.874}} = 241 \text{ mm}$$

Keep  $d = 240 \text{ mm}$  and total thickness = 300 mm so that an effective cover of 60 mm is available. Reduce the total thickness to 200 mm at the top so that effective depth of 140 mm is available at the top.

### 5. Stability of wall

Fully dimensioned wall is shown in Fig. 18.16.

Let  $W_1$  = weight of rectangular portion of stem

$W_2$  = weight of triangular portion of stem

$W_3$  = weight of base slab

$W_4$  = weight of soil on heel slab.

The calculations are arranged in Table 18.1

TABLE 18.1

S. No.	Designation	Force (kN)	Lever arm (m)	Moment about toe (kN-m)
1	$W_1$	$1 \times 0.2 \times 3.7 \times 25 = 18.5$	1.1	20.35
2	$W_2$	$\frac{1}{2} \times 0.1 \times 3.7 \times 25 = 4.63$	0.97	4.49
3	$W_3$	$1 \times 2.4 \times 0.3 \times 25 = 18.0$	1.2	21.60
4	$W_4$	$1 \times 1.2 \times 3.7 \times 18 = 79.97$	1.8	143.86
		$\Sigma W = 121.05$		$M_R = 190.30$

The total resisting moment  $M_R = 190.30 \text{ kN-m}$  ... (1)

$$\begin{aligned} \text{Earth pressure } P &= K_a \gamma \frac{H^2}{2} \\ &= \frac{1}{3} \times \frac{18}{2} (4)^2 = 48 \text{ kN} \end{aligned} \quad \dots (2)$$

### Overturning

$$\text{Overturning moment } M_o = 48 \times \frac{4}{3} = 64 \text{ kN-m} \quad \dots (3)$$

$\therefore$  F.S. against overturning  $= \frac{190.3}{64} = 2.97 > 2$ . Hence safe.

### Sliding

$$\begin{aligned} \text{F.S. against sliding} &= \frac{\mu \Sigma W}{P} \\ &= \frac{0.5 \times 121.05}{48} = 1.26 < 1.5 \end{aligned}$$

Hence unsafe.

To make it safe against sliding, we will have to provide a shear key.

### Pressure distribution

$$\text{Net moment } \Sigma M = 190.3 - 64 = 126.3 \text{ kN-m}$$

$\therefore$  Distance  $\bar{x}$  of the point of application of resultant, from toe is

$$\bar{x} = \frac{\Sigma M}{\Sigma W} = \frac{126.3}{122.05} = 1.04 \text{ m}$$

$$\text{Eccentricity } e = \frac{b}{2} - \bar{x} = 1.2 - 1.04 = 0.16 \text{ m}$$

$$\text{This is less than } \frac{b}{6} \left( = \frac{2.4}{6} = 0.4 \text{ m} \right)$$

$$\begin{aligned} \text{Pressure } p_1 \text{ at toe} &= \frac{\Sigma W}{b} \left( 1 + \frac{6e}{b} \right) \\ &= \frac{121.05}{2.4} \left( 1 + \frac{6 \times 0.16}{2.4} \right) \\ &= 70.61 \text{ kN/m}^2 < 100. \text{ Hence safe.} \end{aligned}$$

$$\begin{aligned} \text{Pressure } p_2 \text{ at heel} &= \frac{\Sigma W}{b} \left( 1 - \frac{6e}{b} \right) \\ &= \frac{121.05}{2.4} \left( 1 - \frac{6 \times 0.16}{2.4} \right) \\ &= 30.26 \text{ kN/m}^2 \end{aligned}$$

Pressure  $p$  at the junction of stem with toe slab is

$$p = 70.61 - \frac{70.61 - 30.26}{2.4} \times 0.9 = 55.48 \text{ kN/m}^2$$

Pressure  $p'$  at the junction of stem with heel slab is

$$p' = 70.61 - \frac{70.61 - 30.26}{2.4} \times 1.2 = 50.44 \text{ kN/m}^2$$

### 6. Design of toe slab

The upward pressure distribution on the toe slab is shown in Fig. 18.17. The weight of the soil above the toe slab is neglected. Thus two forces are acting on it :

(i) upward soil pressure.

(ii) downward weight of slab.

$$\begin{aligned} \text{Downward weight of slab per} \\ \text{unit area} &= 0.3 \times 1 \times 1 \times 25 \\ &= 7.5 \text{ kN/m}^2 \end{aligned}$$

Hence net pressure intensities will be  $= 70.61 - 7.5 = 63.11 \text{ kN/m}^2$  under  $D$  and  $55.48 - 7.5 = 47.98 \text{ kN/m}^2$  under  $E$ .

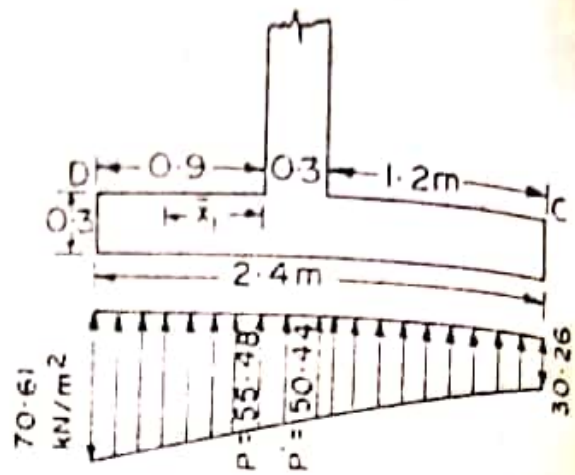


FIG. 18.17.

$$\begin{aligned} \text{Total force} &= \text{S.F. at } E = \\ &= \frac{1}{2}(63.11 + 47.98)0.9 = 50 \text{ kN} \end{aligned}$$

$$\bar{x} \text{ from } E = \left( \frac{47.98 + 2 \times 63.11}{47.98 + 63.11} \right) \frac{0.9}{3} = 0.47 \text{ m.}$$

$$\therefore \text{B.M. at } E = 50 \times 0.47 = 23.52 \text{ kN}\cdot\text{m} = 23.52 \times 10^6 \text{ N}\cdot\text{mm}$$

$$\therefore d = \sqrt{\frac{23.52 \times 10^6}{1000 \times 0.874}} = 164 \text{ mm}$$

Let us keep total depth  $= 260 \text{ mm}$  and effective depth of  $200 \text{ mm}$  so that  $60 \text{ mm}$  effective cover is available. Thickness can be reduced to  $200 \text{ mm}$  at the edge.

$$A_n = \frac{23.52 \times 10^6}{140 \times 0.865 \times 200} = 971 \text{ mm}^2$$

This reinforcement has to be provided at the bottom face. If alternate bars of stem reinforcement are bent and continued in the toe slab, area available  $= \frac{1}{2} \times 2010 = 1005 \text{ mm}^2$  (see step 8). This reinforcement will consist of  $16 \phi$  bars @  $200 \text{ mm c/c}$ . Let us check this reinforcement for development length.

$$L_d \approx 58.3\phi = 58.3 \times 16 = 933 \text{ mm.}$$

Providing  $50 \text{ mm}$  clear side cover, actual length available  $= 900 - 50 +$  anchorage value of hook  $= 850 + 13 \times 16 = 850 + 208 = 1058 > L_d$ . Hence safe.

$$\begin{aligned} \text{Distribution reinforcement} &= \frac{0.15}{100} \times 1000 \left[ \frac{260 + 200}{2} \right] \\ &= 345 \text{ mm}^2 \end{aligned}$$

Using  $8 \text{ mm } \phi$  bars,  $A_\phi = 50.3 \text{ mm}^2$

$$\therefore \text{Spacing} = \frac{1000 \times 50.3}{345} = 146 \text{ mm}$$

Hence provide these @  $140 \text{ mm c/c}$ .

7. *Design of heel slab*

Three forces act on it : (i) downward weight of soil 3.7 m high  
 (ii) downward weight of heel slab  
 (iii) upward soil pressure

$$\text{Total weight of soil} = 1.2 \times 3.7 \times 1 \times 18 = 80 \text{ kN}$$

acting at 0.6 m from B.

$$\text{Total weight of heel slab} = 1.2 \times 0.26 \times 1 \times 25 = 7.8$$

acting at 0.6 m from B.

$$\text{Total upward soil reaction} = \frac{1}{2}(50.44 + 30.26)1.2 = 48.42 \text{ kN}$$

$$\text{acting at } \left( \frac{50.44 + 2 \times 30.26}{50.44 + 30.26} \right) \frac{1.2}{3} = 0.85 \text{ m from B}$$

$$\therefore \text{Total force} = \text{S.F. at B} = 80 + 7.8 - 48.42 = 39.38 \text{ kN}$$

$$\therefore \text{B.M. at B} = (80 \times 0.6) + (7.8 \times 0.6) - (40.58 \times 0.85)$$

$$= 18.19 \text{ kN-m} = 18.19 \times 10^6 \text{ N-mm}$$

$$\therefore d = \sqrt{\frac{18.19 \times 10^6}{1000 \times 0.874}} = 144 \text{ mm.}$$

However keep the same total depth (=260 mm) as that of toe slab, so that available effective depth = 200 mm. The thickness is reduced to 200 mm at the edge.

$$A_{st} = \frac{18.19 \times 10^6}{140 \times 0.864 \times 200} = 752 \text{ mm}^2$$

Using 12 mm  $\phi$  bars,  $A_{\phi} = 113 \text{ mm}^2$

$$\therefore \text{Spacing} = \frac{1000 \times 113}{752} \approx 150 \text{ mm.}$$

Hence provide these @ 150 mm c/c. Take the reinforcement into the toe slab from a distance of  $58.3 \phi = 58.3 \times 12 = 700 \text{ mm}$  to the left of B, and its ends should be hooked.

$$\text{Distribution steel} = \frac{0.15}{100} \times 1000 \left[ \frac{260 + 200}{2} \right] = 345 \text{ mm}^2$$

$$\therefore \text{Spacing} = \frac{1000 \times 50.3}{345} = 146 \text{ mm.}$$

Hence provide three @ 140 mm c/c.

$$\text{Shear stress } \tau_v = \frac{39.38 \times 1000}{1000 \times 200} = 0.20 \text{ N/mm}^2$$

This is much less than the permissible shear stress even at the minimum percentage of steel (Table 3.1).

8. *Reinforcement in the stem*

We had earlier assumed the thickness of heel slab as 0.3 m, while it has now been fixed as 0.26 m only.

Hence revised  $H_1 = 4 - 0.26 = 3.74$  m

$$\begin{aligned}\therefore M &= K_{\alpha} \gamma \frac{H_1^3}{6} = \frac{1}{3} \times \frac{18}{6} (3.74)^3 \\ &= 52.31 \text{ kN-m} = 52.31 \times 10^6 \text{ N-mm}\end{aligned}$$

$$\therefore d = \sqrt{\frac{52.31 \times 10^6}{1000 \times 0.874}} = 245 \text{ mm.}$$

Keep  $d = 250$  mm so that  $D = 310$  mm. Reduce the total thickness at the top.

$$A_{st} = \frac{52.31 \times 10^6}{140 \times 0.865 \times 250} = 1728 \text{ mm}^2$$

Using 16 mm  $\Phi$  bars,  $A_{\Phi} = 201 \text{ mm}^2$

$$\therefore s = \frac{1000 \times 201}{1728} = 116 \text{ mm}$$

However provide 16 mm  $\Phi$  bars @ 100 mm c/c.

$$\text{Actual } A_{st} \text{ provided} = 1000 \times \frac{201}{100} = 2010 \text{ mm}^2.$$

Continue alternate bars in the toe slab to serve as tensile reinforcement there. Discontinue the remaining half bars after a distance of  $58.3\Phi = 58.3 \times 16 \approx 930$  mm beyond  $B$ , in the toe slab.

Between  $A$  and  $B$ , some of the bars can be curtailed. Consider a section at depth  $h$  below the top of the stem. The effective depth  $d'$  at that section is

$$d' = 140 + \frac{250 - 140}{3.74} h = (140 + 29.4h) \text{ mm} \quad \dots(1)$$

where  $h$  is in metres.

$$\text{Now } A_{st} \propto \frac{H^3}{d}$$

$$\text{or } H = (A_{st} d)^{1/3}$$

$$\text{Hence } \frac{h}{H_1} = \left( \frac{A_{st}' d'}{A_{st} d} \right)^{1/3} \quad \dots(2)$$

where  $A_{st}' =$  reinforcement at depth  $h$

$d' =$  effective depth at depth  $h$

$A_{st} =$  reinforcement at depth  $H_1$

$d =$  effective depth at depth  $H_1$

$$\text{If } A_{st}' = \frac{1}{2} A_{st}, \frac{A_{st}'}{A_{st}} = \frac{1}{2}$$

$$\therefore \frac{h}{H_1} = \left( \frac{1}{2} \cdot \frac{d'}{d} \right)^{1/3}$$

Substituting  $d = 250$  mm and  $d' = (140 + 29.4 h)$ , we get

$$h = H_1 \left[ \frac{140 + 29.4 h}{2 \times 250} \right]^{\frac{1}{3}} = 3.74 \left[ \frac{140 + 29.4 h}{500} \right]^{\frac{1}{3}}$$

or  $h = 0.471 [140 + 29.4 h]^{\frac{1}{3}}$

This can be solved by trial and error, noting that if the effective thickness of stem were constant,  $h$  would have been equal to

$$\frac{H_1}{(2)^{\frac{1}{3}}} \approx 0.79 H_1 \approx 2.96 \text{ m.}$$

Solving (3) by trial, we get  $h = 2.86$  m.

Thus, half the bars can be curtailed at this point. However, the bars should be extended by a distance of  $12\phi$  ( $= 12 \times 16 = 192$  mm) or  $d$  ( $= 250$  mm) whichever is more, beyond the point.

$\therefore h = 2.86 - 0.25 = 2.61$  m. Hence curtail half the bars at a height 2.6 m below the top.

If we wish to curtail half of the remaining bars so that remaining reinforcement is one-fourth of that provided at  $B$ , we have

$$\frac{A_{s'}}{A_s} = \frac{1}{4}$$

Hence from (2),  $\frac{h}{H_1} = \left( \frac{1}{4} \frac{d'}{d} \right)^{\frac{1}{3}}$

or  $h = H_1 \left[ \frac{140 + 29.4 h}{4 \times 250} \right]^{\frac{1}{3}} = 3.74 \left[ \frac{140 + 29.4 h}{1000} \right]^{\frac{1}{3}}$   
 $= 0.374 [140 + 29.4 h]^{\frac{1}{3}} \dots(4)$

This can be solved by trial and error, noting that if the effective thickness of stem were constant,  $h$  would have been equal to

$$\frac{H_1}{(4)^{\frac{1}{3}}} = 0.63 H_1 \approx 2.36 \text{ m.}$$

Solving (4) by trial and error, we get  $h = 2.2$  m. However the bars should be extended by 250 mm beyond this.

$$\therefore h = 2.2 - 0.25 = 1.95$$

Hence stop half the remaining bars by 1.95 m below the top of the stem. Continue rest of the bars to the top of the step.

*Check for shear*

$$\begin{aligned} \text{Shear force} = p &= K_s \gamma \frac{H^2}{2} = \frac{1}{3} \times \frac{18}{2} (3.74)^2 \\ &= 41.96 \text{ kN} \end{aligned}$$

$$\therefore \tau_v = \frac{41.96 \times 1000}{1000 \times 250} = 0.17 \text{ N/mm}^2 < \tau_c$$

Hence safe.

### *Distribution and temperature reinforcement*

$$\text{Average thickness of stem} = \frac{1}{2}(310 + 200) = 255 \text{ mm}$$

$$\therefore \text{Distribution reinforcement} = \frac{0.15}{100} \times 1000 \times 255 = 383 \text{ mm.}$$

$$\text{Using } 8 \text{ mm } \phi \text{ bars, } A_\phi = 50.3 \text{ mm}^2$$

$$\therefore \text{Spacing} = \frac{1000 \times 50.3}{383} = 131 \text{ mm}$$

Hence provide 8 mm  $\phi$  bars @ 130 mm c/c at the inner face of the wall, along its length.

For temperature reinforcement, provide 8 mm  $\phi$  @ 260 mm c/c both, ways, in the outer face.

### *9. Design of shear key*

The wall is unsafe in sliding, and hence shear key will have to be provided below the stem, as shown in Fig. 18.18.

Let the depth of key =  $a$ .

Intensity of passive pressure  $p_p$  developed in front of the key depends upon the soil pressure  $p$  in front of the key.

$$\therefore p_p = K_p p = 3 \times 55.48 = 166.4 \text{ kN/m}^2$$

$$\therefore \text{Total passive pressure } P_p = p_p a = 166.4 a \quad \dots(1)$$

Sliding force at level  $D_1C_1$

$$= \frac{1}{3} \times \frac{18}{2} (4 + a)^2$$

$$\text{or } P_H = 3 (4 + a)^2 \quad \dots(2)$$

Weight of soil between bottom of base and  $D_1C_1$

$$= 2.4 a \times 18 = 43.2 a$$

$$\therefore \Sigma W = 121.05 + 43.2 a \quad (\text{Refer Table 18.1})$$

Hence for equilibrium of the wall, permitting F.S. = 1.5 against sliding, we have

$$1.5 = \frac{\mu \Sigma W + P_p}{P_H}$$



or

$$1.5 = \frac{0.5 (121.05 + 43.2 a) + 166.4 a}{3 (4 + a)^2}$$

or

$$a^2 - 33.8 a + 2.54 = 0$$

which gives  $a \approx 0.09 \text{ m} = 90 \text{ mm}$

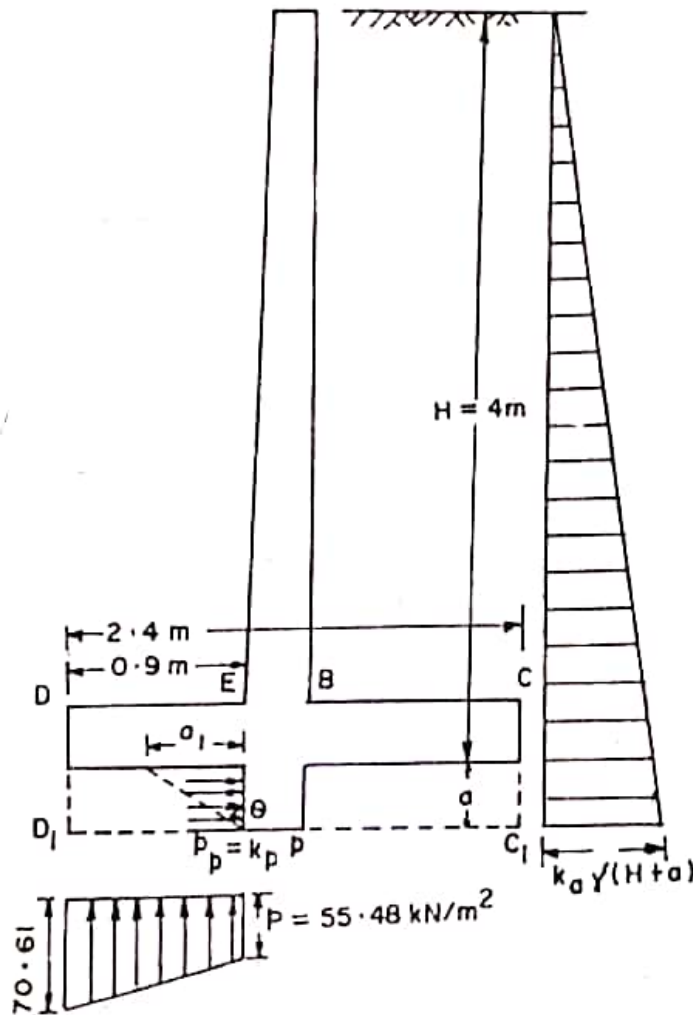


FIG. 18.18.

However, provide a minimum value of  $a = 0.3 \text{ m} = 300 \text{ mm}$ . Keep width of key =  $300 \text{ mm}$ . It should be noted that passive pressure taken into account above will be developed *only when* a length  $a_1$  given below is available in front of the key :

$$\begin{aligned} a_1 &= a \tan \theta = a \tan \left( 45^\circ + \frac{\Phi}{2} \right) \\ &= a \sqrt{K_p}, \text{ where } \left( 45^\circ + \frac{\Phi}{2} \right) \\ &= \text{shearing angle of passive resistance.} \end{aligned}$$

$\therefore a_1 = 0.3 \sqrt{3} = 0.52 \text{ m.}$

Actual length of the slab available =  $DE = 0.9 \text{ m.}$  Hence satisfactory.

Now size of key =  $300 \text{ mm} \times 300 \text{ mm}$

$P_H = 3(4 + a)^2 = 3(4 + 0.3)^2 = 55.47 \text{ kN}$

$P_P = 166.4 a = 166.4 \times 0.3 = 49.92 \text{ kN}$

$\therefore \Sigma W = 121.05 + 43.2 a = 121.05 + 43.2 \times 0.3 = 134.01 \text{ kN}$

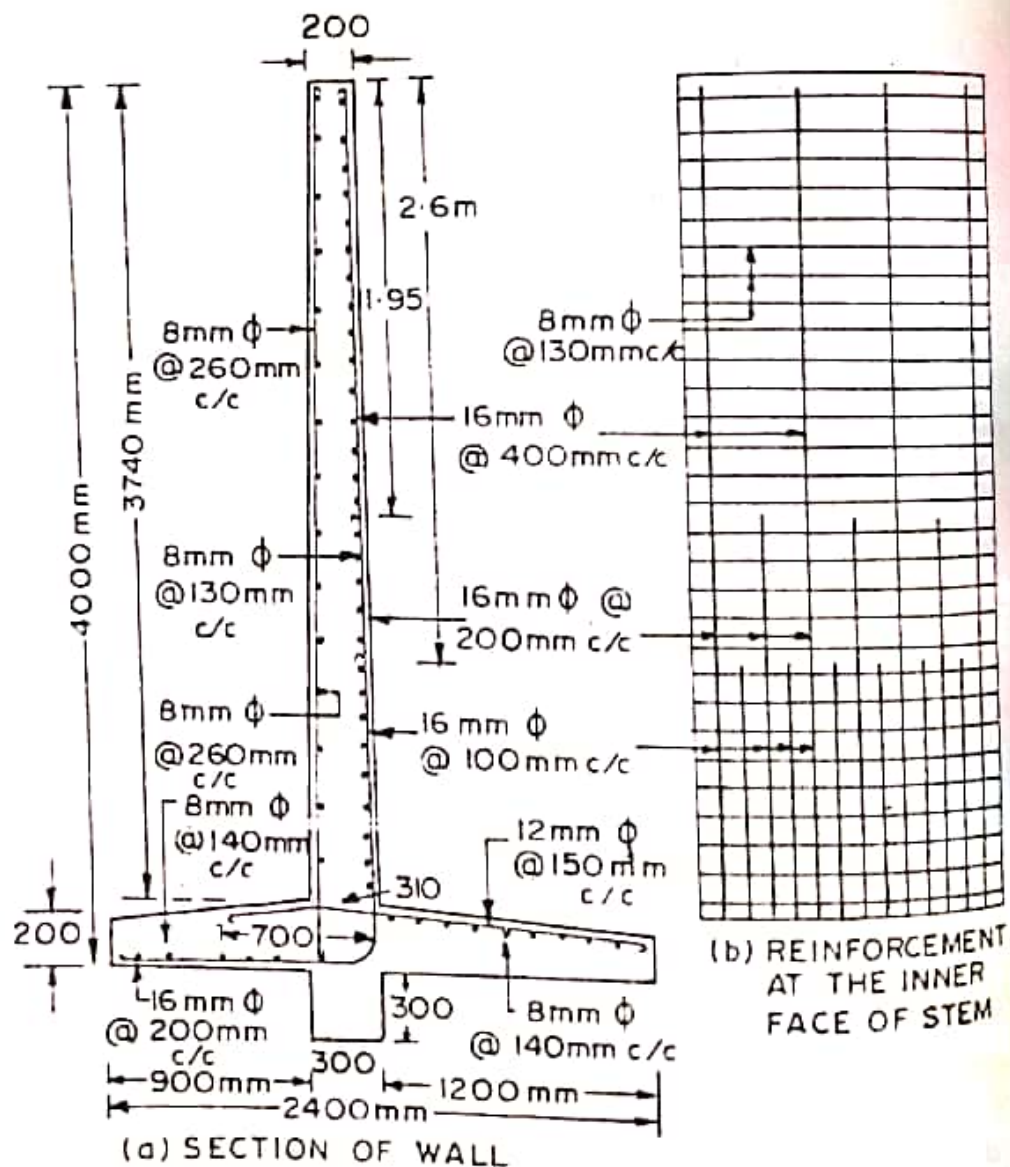


FIG. 18.19.

Actual force to be resisted by the key, at F.S. = 1.5 is

$= 1.5 P_H - \mu \Sigma W$

$= 1.5 \times 55.47 - 0.5 \times 134.01 = 16.2 \text{ kN}$

$$\therefore \text{Shear stress} = \frac{16.2 \times 1000}{300 \times 1000} = 0.054 \text{ N/mm}^2 \text{ (safe)}$$

$$\text{Bending stress} = \frac{16.2 \times 150 \times 1000}{\frac{1}{6} \times 1000 (300)^2} = 0.16 \text{ N/mm}^2 \text{ (safe)}$$